1. Romberg integration gives $R_{3,3}$ as follows:
   (a) 0.1922593
   (b) 0.1606105
   (c) −0.1768200
   (d) 0.08875677
   (e) 2.5879685
   (f) −0.7341567
   (g) 0.6362135
   (h) 0.6426970

2. Romberg integration gives $R_{3,3}$ as follows:
   (a) 1.45281435
   (b) 0.32795861
   (c) −10.51261013
   (d) 0.52681555

3. Romberg integration gives $R_{4,4}$ as follows:
   (a) 0.1922594
   (b) 0.1606028
   (c) −0.1768200
   (d) 0.08875528
   (e) 2.5886272
   (f) −0.7339728
   (g) 0.6362134
   (h) 0.6426991

4. Romberg integration gives $R_{4,4}$ as follows:
   (a) 1.45466031
   (b) 0.32456706
   (c) −10.52012212
   (d) 0.52659385

5. Romberg integration gives:

   (a) 0.19225936 with $n = 4$
   (b) 0.16060279 with $n = 5$
   (c) −0.17682002 with $n = 4$
   (d) 0.088755284 with $n = 5$
   (e) 2.5886286 with $n = 6$
   (f) −0.73396918 with $n = 6$
   (g) 0.63621335 with $n = 4$
   (h) 0.64269908 with $n = 5$

6. (a) $R_{6,6} = 1.45464871$, Actual Integral= 1.454648713
   (b) $R_{7,7} = 0.32433216$, Actual Integral= 0.3243321549
   (c) $R_{6,6} = -10.52001521$, Actual Integral= −10.52001520
   (d) $R_{6,6} = 0.52658903$, Actual Integral= 0.5265890342

7. $R_{33} = 11.5246$

8. $R_{21} = 0.2361$

9. $f(2.5) \approx 0.43457$

10. $f(1/2) = 5.5$

11. $R_{31} = 5$

12. Romberg integration gives:

   (a) 62.4373714, 57.2885616, 56.4437507, 56.2630547, and 56.2187727 yields a prediction of 56.2.
   (b) 55.5722917, 56.2014707, 56.2055989, and 56.2040624 yields a prediction of 56.20.
   (c) 58.3626837, 59.0773207, 59.2688746, 59.3175220, 59.3297316, and 59.3327870 yields a prediction of 59.330.
   (d) 58.4220930, 58.4707174, 58.4704791, and 58.4704691 yields a prediction of 58.47047.
   (e) Consider the graph of the function.

13. We have

$$
\begin{aligned}
R_{k,2} &= \frac{4R_{k,1} - R_{k-1,1}}{3} = \frac{1}{3}\left[R_{k-1,1} + 2h_{k-1}\sum_{i=1}^{2^{k-2}} f(a + (i - 1/2))h_{k-1})\right], \\
&= \frac{1}{3}\left[\frac{h_{k-1}}{2}(f(a) + f(b)) + h_{k-1}\sum_{i=1}^{2^{k-2}-1} f(a + ih_{k-1}) + 2h_{k-1}\sum_{i=1}^{2^{k-2}} f(a + (i - 1/2)h_{k-1})\right].
\end{aligned}
$$

Hence

$$
\begin{aligned}
R_{k,2} &= \frac{1}{3}\left[h_k(f(a) + f(b)) + 2h_k\sum_{i=1}^{2^{k-2}-1} f(a + 2ih_k) + 4h_k\sum_{i=1}^{2^{k-2}} f(a + (2i - 1)h)\right] \\
&= \frac{h}{3}\left[f(a) + f(b) + 2\sum_{i=1}^{M-1} f(a + 2ih) + 4\sum_{i=1}^{M} f(a + (2i - 1)h)\right],
\end{aligned}
$$

where $h = h_k$ and $M = 2^{k-2}$.

14. First consider

$$
\begin{aligned}
\sum_{i=1}^{2N-1} g(i) &= g(1) + g(2) + g(3) + \cdots + g(2N - 2) + g(2N - 1) \\
&= [g(1) + g(3) + \cdots + g(2N - 1)] + [g(2) + g(4) + \cdots + g(2N - 2)] \\
&= \sum_{i=1}^{N} g(2i - 1) + \sum_{i=1}^{N-1} g(2i).
\end{aligned}
$$

The result follows by setting

$$
g(i) = f\left(a + \frac{i}{2}h_{k-1}\right) \quad \text{and} \quad N = 2^{k-2}.
$$

15. Equation (4.34) follows from

$$\begin{aligned}
R_{k,1} &= \frac{h_k}{2}\left[f(a)+f(b)+2\sum_{i=1}^{2^{k-1}-1} f(a+ih_k)\right]\\
&= \frac{h_k}{2}\left[f(a)+f(b)+2\sum_{i=1}^{2^{k-1}-1} f(a+\frac{i}{2}h_{k-1})\right]\\
&= \frac{h_k}{2}\left[f(a)+f(b)+2\sum_{i=1}^{2^{k-1}-1} f(a+ih_{k-1})+2\sum_{i=1}^{2^{k-2}} f(a+(i-1/2)h_{k-1})\right]\\
&= \frac{1}{2}\left\{\frac{h_{k-1}}{2}\left[f(a)+f(b)+2\sum_{i=1}^{2^{k-2}-1} f(a+ih_{k-1})\right]+h_{k-1}\sum_{i=1}^{2^{k-2}} f(a+(i-1/2)h_{k-1})\right\}\\
&= \frac{1}{2}\left[R_{k-1,1}+h_{k-1}\sum_{i=1}^{2^{k-2}} f(a+(i-1/2)h_{k-1})\right].
\end{aligned}$$

16. The approximation $\mathrm{erf}(1)\approx 0.84270079$ is obtained using $n=6$.