1. The parametric cubic Hermite approximations are as follows.

   (a) $x(t) = -10t^3 + 14t^2 + t, \quad y(t) = -2t^3 + 3t^2 + t$
   (b) $x(t) = -10t^3 + 14.5t^2 + 0.5t, \quad y(t) = -3t^3 + 4.5t^2 + 0.5t$
   (c) $x(t) = -10t^3 + 14t^2 + t, \quad y(t) = -4t^3 + 5t^2 + t$
   (d) $x(t) = -10t^3 + 13t^2 + 2t, \quad y(t) = 2t$

2. The parametric cubic Bézier approximations are as follows.

   (a) $x(t) = -10t^3 + 12t^2 + 3t, \quad y(t) = 2t^3 - 3t^2 + 3t$
   (b) $x(t) = -10t^3 + 13.5t^2 + 1.5t, \quad y(t) = -t^3 + 1.5t^2 + 1.5t$
   (c) $x(t) = -10t^3 + 12t^2 + 3t, \quad y(t) = -4t^3 + 3t^2 + 3t$
   (d) $x(t) = -10t^3 + 9t^2 + 6t, \quad y(t) = 8t^3 - 12t^2 + 6t$

3. The parametric cubic Bézier approximations are as follows.

   (a) $x(t) = -11.5t^3 + 15t^2 + 1.5t + 1, \quad y(t) = -4.25t^3 + 4.5t^2 + 0.75t + 1$
   (b) $x(t) = -6.25t^3 + 10.5t^2 + 0.75t + 1, \quad y(t) = -3.5t^3 + 3t^2 + 1.5t + 1$
   (c) For t between $(0,0)$ and $(4,6)$, we have

   $$x(t) = -5t^3 + 7.5t^2 + 1.5t, \quad y(t) = -13.5t^3 + 18t^2 + 1.5t,$$

   and for t between $(4,6)$ and $(6,1)$, we have

   $$x(t) = -5.5t^3 + 6t^2 + 1.5t + 4, \quad y(t) = 4t^3 - 6t^2 - 3t + 6.$$

   (d) For t between $(0,0)$ and $(2,1)$, we have

   $$x(t) = -5.5t^3 + 6t^2 + 1.5t, \quad y(t) = -0.5t^3 + 1.5t,$$

   for t between $(2,1)$ and $(4,0)$, we have

   $$x(t) = -4t^3 + 3t^2 + 3t + 2, \quad y(t) = -t^3 + 1,$$

   and for t between $(4,0)$ and $(6,-1)$, we have

   $$x(t) = -8.5t^3 + 13.5t^2 - 3t + 4, \quad y(t) = -3.25t^3 + 5.25t^2 - 3t.$$

4. Between $(3,6)$ and $(2,2)$, we have

   $$x(t) = 0.5t^3 - 2.4t^2 + 0.9t + 3, \quad y(t) = 6.5t^3 - 12t^2 + 1.5t + 6;$$

   between $(2,2)$ and $(6,6)$, we have

   $$x(t) = -5.9t^3 + 8.4t^2 + 1.5t + 2, \quad y(t) = -3.5t^3 + 6t^2 + 1.5t + 2;$$

   between $(6,6)$ and $(5,2)$, we have

   $$x(t) = -2.5t^3 + 4.5t^2 - 3t + 6, \quad y(t) = 6.8t^3 - 10.2t^2 - 0.6t + 6;$$

   and between $(5,2)$ and $(6.5,3)$, we have

   $$x(t) = -4.2t^3 + 7.2t^2 - 1.5t + 5, \quad y(t) = 0.1t^3 - 0.6t^2 + 1.5t + 2.$$

5. (a) Using the forward divided difference gives the following table.

0	u_0			
0	u_0	$3(u_1 - u_0)$		
1	u_3	$u_3 - u_0$	$u_3 - 3u_1 + 2u_0$	
1	u_3	$3(u_3 - u_2)$	$2u_3 - 3u_2 + u_0$	$u_3 - 3u_2 + 3u_1 - u_0$

Therefore,

$$\begin{aligned} u(t) &= u_0 + 3(u_1 - u_0)t + (u_3 - 3u_1 + 2u_0)t^2 + (u_3 - 3u_2 + 3u_1 - u_0)t^2(t-1) \\ &= u_0 + 3(u_1 - u_0)t + (-6u_1 + 3u_0 + 3u_2)t^2 + (u_3 - 3u_2 + 3u_1 - u_0)t^3. \end{aligned}$$

Similarly, $v(t) = v_0 + 3(v_1 - v_0)t + (3v_2 - 6v_1 + 3v_0)t^2 + (v_3 - 3v_2 + 3v_1 - v_0)t^3$.

(b) Using the formula for Bernstein polynomials gives, for f,

$$\begin{aligned} \sum_{k=0}^{3} \binom{3}{k} u_k t^k (1-t)^{3-k} &= u_0(1-t)^3 + 3u_1 t(1-t)^2 + 3u_2 t^2(1-t) + u_3 t^3 \\ &= u_0 + 3(u_1 - u_0)t + (3u_2 - 6u_1 + 3u_0)t^2 + (u_3 - 3u_2 + 3u_1 - u_0)t^3 \\ &= u(t) \end{aligned}$$

and, for g,

$$\begin{aligned} \sum_{k=0}^{3} \binom{3}{k} v_k t^k (1-t)^{3-k} &= v_0(1-t)^3 + 3v_1 t(1-t)^2 + 3v_2 t^2(1-t) + v_3 t^3 \\ &= v_0 + 3(v_1 - v_0)t + (3v_2 - 6v_1 + 3v_0)t^2 + (v_3 - 3v_2 + 3v_1 - v_0)t^3 \\ &= v(t). \end{aligned}$$