1. The coefficients for the polynomials in divided-difference form are given in the following tables. For example, the polynomial in part (a) is

$$H_3(x) = 17.56492 + 3.116256(x - 8.3) + 0.05948(x - 8.3)^2 - 0.00202222(x - 8.3)^2(x - 8.6).$$

(a)	(b)	(c)	(d)
17.56492	0.22363362	−0.02475	−0.62049958
3.116256	2.1691753	0.751	3.5850208
0.05948	0.01558225	2.751	−2.1989182
−0.00202222	−3.2177925	1	−0.490447
		0	0.037205
		0	0.040475
			−0.0025277777
			0.0029629628

2. The coefficients for the polynomials in divided-difference form are given in the following tables. For example, the polynomial in part (a) is $H_3(x) = 1 + 2x + 2.87312x^2 + 2.25376x^2(x - 0.5)$

(a)	(b)	*(c)	(d)
1.0	1.33203	−0.29004996	0.8619948
2.0	0.4375	−2.8019975	0.1553624
2.87312	−2.999996	0.945237	0.07337636
2.25376	7.749984	−0.297	0.01583112
		−0.47935	−0.00014728
		0.05	−0.00089244
			−0.00007672
			0.00005975111111

3. The following table shows the approximations.

	x	Approximation to $f(x)$	Actual $f(x)$	Error
(a)	8.4	17.877144	17.877146	2.33×10^{-6}
(b)	0.9	0.44392477	0.44359244	3.3323×10^{-4}
(c)	$-\frac{1}{3}$	0.1745185	0.17451852	1.85×10^{-8}
(d)	0.25	−0.1327719	−0.13277189	5.42×10^{-9}

4. The following table shows the approximations.

	x	Approximation to $f(x)$	Actual $f(x)$	Error
(a)	0.43	2.362069472	2.363160694	0.001091222
(b)	0.0	1.132811175	1.000000000	0.132811750
(c)	0.18	-0.5081234697	-0.5081234644	0.53×10^{-8}
(d)	0.25	1.189069883	1.189069931	0.48×10^{-7}

5. (a) We have $\sin 0.34 \approx H_5(0.34) = 0.33349$.

   (b) The formula gives an error bound of 3.05×10^{-14}, but the actual error is 2.91×10^{-6}. The discrepancy is due to the fact that the data are given to only five decimal places.

   (c) We have $\sin 0.34 \approx H_7(0.34) = 0.33350$. Although the error bound is now 5.4×10^{-20}, the accuracy of the given data dominates the calculations. This result is actually less accurate than the approximation in part (b), since $\sin 0.34 = 0.333487$.

6. (a) $H(1.03) = 0.80932485$. The actual error is 1.24×10^{-6}, and error bound is 1.31×10^{-6}.

   (b) $H(1.03) = 0.809323619263$. The actual error is 3.63×10^{-10}, and an error bound is 3.86×10^{-10}.

7. For 3(a), we have an error bound of 5.9×10^{-8}. The error bound for 3(c) is 0 since $f^{(n)}(x) \equiv 0$, for $n > 3$.

8. For 4(a), we have an error bound of 1.6×10^{-3}. The error bound for 4(c) is 1.5×10^{-7}.

9. $H_3(1.25) = 1.169080403$ with an error bound of 4.81×10^{-5}, and $H_5(1.25) = 1.169016064$ with an error bound of 4.43×10^{-4}.

10. The Hermite polynomial generated from these data is

$$
\begin{aligned}
H_9(x) =& 75x + 0.222222x^2(x-3) - 0.0311111x^2(x-3)^2 \\
&- 0.00644444x^2(x-3)^2(x-5) + 0.00226389x^2(x-3)^2(x-5)^2 \\
&- 0.000913194x^2(x-3)^2(x-5)^2(x-8) + 0.000130527x^2(x-3)^2(x-5)^2(x-8)^2 \\
&- 0.0000202236x^2(x-3)^2(x-5)^2(x-8)^2(x-13).
\end{aligned}
$$

(a) The Hermite polynomial predicts a position of $H_9(10) = 743$ ft and a speed of $H_9'(10) = 48$ ft/sec. Although the position approximation is reasonable, the low speed prediction is suspect.

(b) To find the first time the speed exceeds 55 mi/hr, which is equivalent to $80.\bar{6}$ ft/sec, we solve for the smallest value of t in the equation $80.\bar{6} = H_9'(x)$. This gives $x \approx 5.6488092$.

(c) The estimated maximum speed is $H_9'(12.37187) = 119.423$ ft/sec ≈ 81.425 mi/hr.

11. (a) Suppose $P(x)$ is another polynomial with $P(x_k) = f(x_k)$ and $P'(x_k) = f'(x_k)$, for $k = 0, ..., n$, and the degree of $P(x)$ is at most $2n+1$. Let

$$D(x) = H_{2n+1}(x) - P(x).$$

Then $D(x)$ is a polynomial of degree at most $2n+1$ with $D(x_k) = 0$, and $D'(x_k) = 0$, for each $k = 0, 1, ..., n$. Thus, D has zeros of multiplicity 2 at each x_k and

$$D(x) = (x-x_0)^2 \ldots (x-x_n)^2 Q(x).$$

Hence, $D(x)$ must be of degree $2n$ or more, which would be a contradiction, or $Q(x) \equiv 0$ which implies that $D(x) \equiv 0$. Thus, $P(x) \equiv H_{2n+1}(x)$.

(b) First note that the error formula holds if $x = x_k$ for any choice of ξ.
Let $x \neq x_k$, for $k = 0, ..., n$, and define

$$g(t) = f(t) - H_{2n+1}(t) - \frac{(t-x_0)^2 \ldots (t-x_n)^2}{(x-x_0)^2 \ldots (x-x_n)^2}\left[f(x) - H_{2n+1}(x)\right].$$

Note that $g(x_k) = 0$, for $k = 0, ..., n$, and $g(x) = 0$. Thus, g has $n+2$ distinct zeros in $[a, b]$. By Rolle's Theorem, g' has $n+1$ distinct zeros $\xi_0, ..., \xi_n$, which are between the numbers $x_0, ..., x_n, x$.
In addition, $g'(x_k) = 0$, for $k = 0, ..., n$, so g' has $2n+2$ distinct zeros $\xi_0, ..., \xi_n, x_0, ..., x_n$. Since g' is $2n+1$ times differentiable, the Generalized Rolle's Theorem implies that a number ξ in$[a, b]$ exists with $g^{(2n+2)}(\xi) = 0$. But,

$$g^{(2n+2)}(t) = f^{(2n+2)}(t) - \frac{d^{2n+2}}{dt^{2n+2}} H_{2n+1}(t) - \frac{[f(x) - H_{2n+1}(x)] \cdot (2n+2)!}{(x-x_0)^2 \cdots (x-x_n)^2}$$

and

$$0 = g^{(2n+2)}(\xi) = f^{(2n+2)}(\xi) - \frac{(2n+2)![f(x) - H_{2n+1}(x)]}{(x-x_0)^2 \cdots (x-x_n)^2}.$$

The error formula follows.

12. Let

$$H(x) = f[z_0] + f[z_0, z_1](x - x_0) + f[z_0, z_1, z_2](x - x_0)^2 + f[z_0, z_1, z_2, z_3](x - x_0)^2(x - x_1).$$

Substituting $f[z_0] = f(x_0)$, $f[z_0, z_1] = f'(x_0)$,

$$f[z_0, z_1, z_2] = \frac{f(x_1) - f(x_0) - f'(x_0)(x_1 - x_0)}{x_1 - x_0},$$

and

$$f[z_0, z_1, z_2, z_3] = \frac{f'(x_1)(x_1 - x_0) - 2f(x_1) + 2f(x_0) + f'(x_0)(x_1 - x_0)}{(x_1 - x_0)^3}$$

into $H(x)$ and simplifying gives

$$\begin{aligned} H(x) =& f(x_0) + f'(x_0)(x - x_0) + \frac{f(x_1) - f(x_0) - f'(x_0)(x_1 - x_0)}{(x_1 - x_0)^2}(x - x_0)^2 \\ &+ \frac{f'(x_1)(x_1 - x_0) - 2f(x_1) + 2f(x_0) + f'(x_0)(x_1 - x_0)}{(x_1 - x_0)^3}(x - x_0)^2(x - x_1). \end{aligned}$$

Thus, $H(x_0) = f(x_0)$ and

$$H(x_1) = f(x_0) + f'(x_0)(x_1 - x_0) + [f(x_1) - f(x_0) - f'(x_0)(x_1 - x_0)] = f(x_1).$$

Further,

$$\begin{aligned} H'(x) =& f'(x_0) + 2\frac{f(x_1) - f(x_0) - f'(x_0)(x_1 - x_0)}{(x_1 - x_0)^2}(x - x_0) \\ &+ \frac{f'(x_1)(x_1 - x_0) - 2f(x_1) + 2f(x_0) + f'(x_0)(x_1 - x_0)}{(x_1 - x_0)^3}[2(x - x_0)(x - x_1) + (x - x_0)^2], \end{aligned}$$

so

$$H'(x_0) = f'(x_0)$$

and

$$\begin{aligned} H'(x_1) =& f'(x_0) + \frac{2f(x_1)}{x_1 - x_0} - \frac{2f(x_0)}{x_1 - x_0} - 2f'(x_0) + f'(x_1) - \frac{2f(x_1)}{x_1 - x_0} + \frac{2f(x_0)}{x_1 - x_0} + f'(x_0) \\ =& f'(x_1). \end{aligned}$$

Thus, H satisfies the requirements of the cubic Hermite polynomial H_3, and the uniqueness of H_3 implies $H_3 = H$.