

1. Interpolation polynomials give the following results.

(a)

n	$x_0, x_1, \dots, x_n$	$P_n(8.4)$
1	8.3, 8.6	17.87833
2	8.3, 8.6, 8.7	17.87716
3	8.3, 8.6, 8.7, 8.1	17.87714

(b)

n	$x_0, x_1, \dots, x_n$	$P_n(-1/3)$
1	-0.5, -0.25	0.21504167
2	-0.5, -0.25, 0.0	0.16988889
3	-0.5, -0.25, 0.0, -0.75	0.17451852

(c)

n	$x_0, x_1, \dots, x_n$	$P_n(0.25)$
1	0.2, 0.3	-0.13869287
2	0.2, 0.3, 0.4	-0.13259734
3	0.2, 0.3, 0.4, 0.1	-0.13277477

(d)

n	$x_0, x_1, \dots, x_n$	$P_n(0.9)$
1	0.8, 1.0	0.44086280
2	0.8, 1.0, 0.7	0.43841352
3	0.8, 1.0, 0.7, 0.6	0.44198500

2. Interpolation polynomials give the following results.

(a) $P_{1,2} = 2.418803200$
 $|f(0.43) - P_{1,2}| = 0.055642506;$
 $P_{1,2,3} = 2.348863120;$
 $|f(0.43) - P_{1,2,3}| = 0.014297574$
 $P_{0,1,2,3} = 2.360604734;$
 $|f(0.43) - P_{0,1,2,3}| = 0.002555960e$

(b) $P_{1,2} = 1.066405500$
 $|f(0.0) - P_{1,2}| = 0.066405500;$
 $P_{0,1,2} = 0.9531236670;$
 $|f(0.0) - P_{0,1,2}| = 0.0468763330$
 $P_{0,1,2,3} = 0.9843740000;$
 $|f(0.0) - P_{0,1,2,3}| = 0.0156260000$

(c) $P_{0,1} = -0.506647844$
 $|f(0.18) - P_{0,1}| = 0.0014756204;$
 $P_{0,1,2} = -0.5080498520;$
 $|f(0.18) - P_{0,1,2}| = 0.0000736124$
 $P_{0,1,2,3} = -0.5081430745;$
 $|f(0.18) - P_{0,1,2,3}| = 0.0000196101$

(d) $P_{2,3} = 1.196494500$
 $|f(0.25) - P_{2,3}| = 0.007424569;$
 $P_{1,2,3} = 1.189597976;$
 $|f(0.25) - P_{1,2,3}| = 0.000528045$
 $P_{0,1,2,3} = 1.188935147;$
 $|f(0.25) - P_{0,1,2,3}| = 0.000134784$

3. (a) We have $\sqrt{3} \approx P_{0,1,2,3,4} = 1.708\overline{3}$.

(b) We have $\sqrt{3} \approx P_{0,1,2,3,4} = 1.690607$.

(c) Absolute error in part (a) is approximately 0.0237, and the absolute error in part (b) is 0.0414, so part (a) is more accurate.

4. The Neville table is

$x_0 = 0.0$	0				
$x_1 = 0.5$	y	$3y$			
$x_2 = 1.0$	3	$6 - y$	$9 - 3y$		
$x_3 = 2.0$	2	2.5	$\frac{11}{3} - \frac{1}{3}y$	$5 - y$	

Since the last difference is 0, $y = 5$.

5. $P_2 = f(0.5) = 4$

6. $P_2 = f(0.7) = 6.4$

7. $P_{0,1,2,3}(2.5) = 2.875$

8. $P_{0,1,2,3}(1.5) = 3.625$

9. The approximation is

$$-\frac{1}{6}f(2) + \frac{2}{3}f(1) + \frac{2}{3} + \frac{2}{3}f(-1) - \frac{1}{6}f(-2),$$

and the correct value is

$$-\frac{1}{6}f(2) + \frac{2}{3}f(1) + \frac{2}{3}f(-1) - \frac{1}{6}f(-2),$$

so the approximation is too large by the amount $\frac{2}{3}$.

10. The approximation is

$$-\frac{1}{6}f(2) + \frac{2}{3}f(1) - \frac{2}{3} + \frac{2}{3}f(-1) - \frac{1}{6}f(-2),$$

and the correct value is

$$-\frac{1}{6}f(2) + \frac{2}{3}f(1) + \frac{2}{3}f(-1) - \frac{1}{6}f(-2),$$

so the approximation is too small by the amount $\frac{2}{3}$.

11. The first ten terms of the sequence are 0.038462, 0.333671, 0.116605, -0.371760 , -0.0548919 , 0.605935, 0.190249, -0.513353 , -0.0668173 , and 0.448335. Since $f(1 + \sqrt{10}) = 0.0545716$, the sequence does not appear to converge.

12. The solution is approximately 0.567142.

13. Change Algorithm 3.1 as follows:

INPUT numbers $y_0, y_1, \dots, y_n$; values $x_0, x_1, \dots, x_n$ as the first column $Q_{0,0}, Q_{1,0}, \dots, Q_{n,0}$ of Q .
OUTPUT the table Q with $Q_{n,n}$ approximating $f^{-1}(0)$.

STEP 1 For $i = 1, 2, \dots, n$
for $j = 1, 2, \dots, i$
set

$$Q_{i,j} = \frac{y_i Q_{i-1,j-1} - y_{i-j} Q_{i,j-1}}{y_i - y_{i-j}}.$$