1. The results are listed in the following table.

	(a)	(b)	(c)	(d)
$\hat{p}_0$	0.258684	0.907859	0.548101	0.731385
$\hat{p}_1$	0.257613	0.909568	0.547915	0.736087
$\hat{p}_2$	0.257536	0.909917	0.547847	0.737653
$\hat{p}_3$	0.257531	0.909989	0.547823	0.738469
$\hat{p}_4$	0.257530	0.910004	0.547814	0.738798
$\hat{p}_5$	0.257530	0.910007	0.547810	0.738958

2. Newton's Method gives $p_6 = -0.1828876$ and $\hat{p}_6 = -0.183387$.

3. Steffensen's method gives $p_0^{(1)} = 0.826427$.

4. Steffensen's method gives $p_0^{(1)} = 2.152905$ and $p_0^{(2)} = 1.873464$.

5. Steffensen's method gives $p_1^{(0)} = 1.5$.

6. Steffensen's method gives $p_2^{(0)} = 1.73205$.

7. For $g(x) = \sqrt{1 + \frac{1}{x}}$ and $p_0^{(0)} = 1$, we have $p_0^{(3)} = 1.32472$.

8. For $g(x) = 2^{-x}$ and $p_0^{(0)} = 1$, we have $p_0^{(3)} = 0.64119$.

9. For $g(x) = 0.5(x + \frac{3}{x})$ and $p_0^{(0)} = 0.5$, we have $p_0^{(4)} = 1.73205$.

10. For $g(x) = \frac{5}{\sqrt{x}}$ and $p_0^{(0)} = 2.5$, we have $p_0^{(3)} = 2.92401774$.

11. (a) For $g(x) = \left(2 - e^x + x^2\right)/3$ and $p_0^{(0)} = 0$, we have $p_0^{(3)} = 0.257530$.
    (b) For $g(x) = 0.5(\sin x + \cos x)$ and $p_0^{(0)} = 0$, we have $p_0^{(4)} = 0.704812$.
    (c) With $p_0^{(0)} = 0.25$, $p_0^{(4)} = 0.910007572$.
    (d) With $p_0^{(0)} = 0.3$, $p_0^{(4)} = 0.469621923$.

12. (a) For $g(x) = 2 + \sin x$ and $p_0^{(0)} = 2$, we have $p_0^{(4)} = 2.55419595$.
    (b) For $g(x) = \sqrt[3]{2x + 5}$ and $p_0^{(0)} = 2$, we have $p_0^{(2)} = 2.09455148$.
    (c) With $g(x) = \sqrt{\frac{e^x}{3}}$ and $p_0^{(0)} = 1$, we have $p_0^{(3)} = 0.910007574$.
    (d) With $g(x) = \cos x$, and $p_0^{(0)} = 0$, we have $p_0^{(4)} = 0.739085133$.

13. Aitken's Δ^2 method gives:

(a) $\hat{p}_{10} = 0.0\overline{45}$

(b) $\hat{p}_2 = 0.0363$

14. (a) A positive constant λ exists with

$$\lambda = \lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha}.$$

Hence

$$\lim_{n\to\infty} \left| \frac{p_{n+1} - p}{p_n - p} \right| = \lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha} \cdot |p_n - p|^{\alpha-1} = \lambda \cdot 0 = 0 \quad \text{and} \quad \lim_{n\to\infty} \frac{p_{n+1} - p}{p_n - p} = 0.$$

(b) One example is $p_n = \frac{1}{n^n}$.

15. We have

$$\frac{|p_{n+1} - p_n|}{|p_n - p|} = \frac{|p_{n+1} - p + p - p_n|}{|p_n - p|} = \left| \frac{p_{n+1} - p}{p_n - p} - 1 \right|,$$

so

$$\lim_{n\to\infty} \frac{|p_{n+1} - p_n|}{|p_n - p|} = \lim_{n\to\infty} \left| \frac{p_{n+1} - p}{p_n - p} - 1 \right| = 1.$$

16.

$$\frac{\hat{p}_n - p}{p_n - p} = \frac{\lambda\,(\delta_n + \delta_{n+1}) - 2\delta_n + \delta_n\delta_{n+1} - 2\delta_n(\lambda - 1) - \delta_n^2}{(\lambda - 1)^2 + \lambda\,(\delta_n + \delta_{n+1}) - 2\delta_n + \delta_n\delta_{n+1}}$$

17. (a) Since $p_n = P_n(x) = \sum_{k=0}^{n} \frac{1}{k!}x^k$, we have

$$p_n - p = P_n(x) - e^x = \frac{-e^\xi}{(n+1)!}x^{n+1},$$

where ξ is between 0 and x. Thus, $p_n - p \neq 0$, for all $n \geq 0$. Further,

$$\frac{p_{n+1} - p}{p_n - p} = \frac{\frac{-e^{\xi_1}}{(n+2)!}x^{n+2}}{\frac{-e^\xi}{(n+1)!}x^{n+1}} = \frac{e^{(\xi_1 - \xi)}x}{n+2},$$

where ξ_1 is between 0 and 1. Thus, $\lambda = \lim_{n\to\infty} \frac{e^{(\xi_1-\xi)}x}{n+2} = 0 < 1$.

(b)

n	p_n	$\hat{p}_n$
0	1	3
1	2	2.75
2	2.5	$2.7\overline{2}$
3	$2.\overline{6}$	2.71875
4	$2.708\overline{3}$	$2.718\overline{3}$
5	$2.71\overline{6}$	2.7182870
6	$2.7180\overline{5}$	2.7182823
7	2.7182539	2.7182818
8	2.7182787	2.7182818
9	2.7182815	
10	2.7182818	

(c) Aitken's Δ^2 method gives quite an improvement for this problem. For example, $\hat{p}_6$ is accurate to within 5×10^{-7}. We need p_{10} to have this accuracy.