1. $p_3 = 0.625$

2. (a) $p_3 = -0.6875$
   (b) $p_3 = 1.09375$

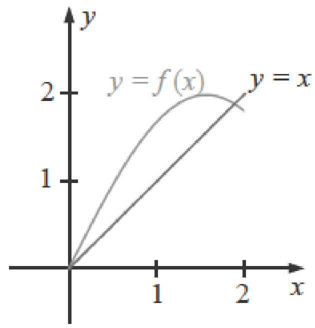
3. The Bisection method gives:
   (a) $p_7 = -1.414$
   (b) $p_8 = 1.414$
   (c) $p_7 = 2.727$
   (d) $p_7 = -0.7265$

4. The Bisection method gives:
   (a) $p_7 = 0.5859$
   (b) $p_8 = 3.002$
   (c) $p_7 = 3.419$

5. The Bisection method gives:
   (a) $p_{17} = 0.641182$
   (b) $p_{17} = 0.257530$
   (c) For the interval $[-3, -2]$, we have $p_{17} = -2.191307$, and for the interval $[-1, 0]$, we have $p_{17} = -0.798164$.
   (d) For the interval $[0.2, 0.3]$, we have $p_{14} = 0.297528$, and for the interval $[1.2, 1.3]$, we have $p_{14} = 1.256622$.

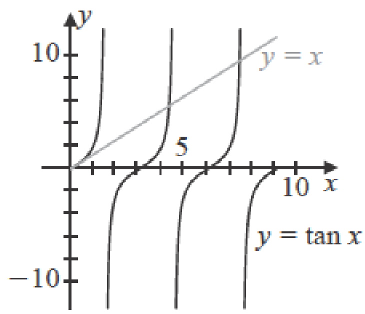
6. (a) $p_{17} = 1.51213837$
   (b) $p_{17} = 0.97676849$
   (c) For the interval $[1, 2]$, we have $p_{17} = 1.41239166$, and for the interval $[2, 4]$, we have $p_{18} = 3.05710602$.
   (d) For the interval $[0, 0.5]$, we have $p_{16} = 0.20603180$, and for the interval $[0.5, 1]$, we have $p_{16} = 0.68196869$.

7. (a)



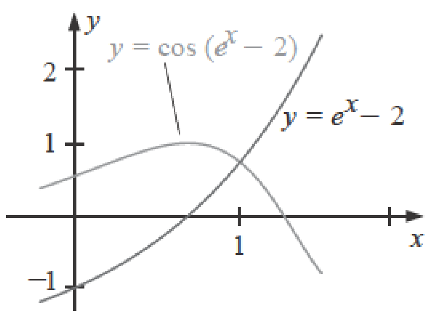
(b) Using [1.5, 2] from part (a) gives $p_{16} = 1.89550018$.

8. (a)



(b) Using [4.2, 4.6] from part (a) gives $p_{16} = 4.4934143$.

9. (a)



(b) $p_{17} = 1.00762177$

10. (a) 0
(b) 0
(c) 2
(d) −2

11. (a) 2

    (b) -2

    (c) -1

    (d) 1

12. We have $\sqrt{3} \approx p_{14} = 1.7320$, using $[1, 2]$.

13. The third root of 25 is approximately $p_{14} = 2.92401$, using $[2, 3]$.

14. A bound for the number of iterations is $n \geq 12$ and $p_{12} = 1.3787$.

15. A bound is $n \geq 14$, and $p_{14} = 1.32477$.

16. For $n > 1$,

$$|f(p_n)| = \left(\frac{1}{n}\right)^{10} \leq \left(\frac{1}{2}\right)^{10} = \frac{1}{1024} < 10^{-3},$$

so

$$|p - p_n| = \frac{1}{n} < 10^{-3} \Leftrightarrow 1000 < n.$$

17. Since $\lim_{n\to\infty}(p_n - p_{n-1}) = \lim_{n\to\infty} 1/n = 0$, the difference in the terms goes to zero. However, p_n is the nth term of the divergent harmonic series, so $\lim_{n\to\infty} p_n = \infty$.

18. Since $-1 < a < 0$ and $2 < b < 3$, we have $1 < a + b < 3$ or $1/2 < 1/2(a + b) < 3/2$ in all cases. Further,

$$\begin{aligned} f(x) < 0, &\quad \text{for } -1 < x < 0 \quad \text{and} \quad 1 < x < 2; \\ f(x) > 0, &\quad \text{for } \ 0 < x < 1 \quad \text{and} \quad 2 < x < 3. \end{aligned}$$

Thus, $a_1 = a$, $f(a_1) < 0$, $b_1 = b$, and $f(b_1) > 0$.

(a) Since $a + b < 2$, we have $p_1 = \frac{a+b}{2}$ and $1/2 < p_1 < 1$. Thus, $f(p_1) > 0$. Hence, $a_2 = a_1 = a$ and $b_2 = p_1$. The only zero of f in $[a_2, b_2]$ is $p = 0$, so the convergence will be to 0.

(b) Since $a + b > 2$, we have $p_1 = \frac{a+b}{2}$ and $1 < p_1 < 3/2$. Thus, $f(p_1) < 0$. Hence, $a_2 = p_1$ and $b_2 = b_1 = b$. The only zero of f in $[a_2, b_2]$ is $p = 2$, so the convergence will be to 2.

(c) Since $a + b = 2$, we have $p_1 = \frac{a+b}{2} = 1$ and $f(p_1) = 0$. Thus, a zero of f has been found on the first iteration. The convergence is to $p = 1$.

19. The depth of the water is 0.838 ft.

20. The angle θ changes at the approximate rate $w = -0.317059$.