Homework Assignment for Week 17, the Last One!

Assigned Jan 06, 2010. Due Jan 15, 2010.

1. Section 7.2: Problems 4, 9, 16, 19, 20, 21.

   For problems 16 and 19, read Example 7.3.4.

   For problem 19, you may assume in addition that A is symmetric. The general case can be proved using the same method, but requires more description on the details.

   For problem 21, just find the eigenvalues and eigenvectors using matlab built-in function 'eig'. Ignore the sensitivity part.

   Extra credit problems: basically due on Jan 15, the final exam day. Short extension may be possible upon request (by Jan 15).

2. Section 7.2: Problems 19 (the general case).

3. The error formula (5.35), as an equality, is not straightforward (try Taylor expansion and you will see). Here is a trick that may be useful in other applications: Define

$$E(u) = \int_{-u}^{u} f(t)dt - \frac{u}{3}[f(-u) + 4f(0) + f(u)],$$

   as a function of the variable u. Obviously, $E(h)$ gives (5.35) with $a = -h$, $b = h$. Try to verify (5.35) with this formula. You probably need the integral form of the Taylor Theorem's remainder term.

4. This is continuation of Problem 3, Homework 08. We knew that

$$\frac{c_1}{n} \le \frac{1}{n}\sum_{i=0}^{n-1} \tan^2(\frac{i\pi}{2n}) \le \frac{C_1}{n}$$

   by using area comparison with the corresponding integral. Try to find

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \tan^2(\frac{i\pi}{2n}),$$

   or at least find c_2 and C_2, with $c_1 < c_2$, $C_2 < C_1$, such that

$$\frac{c_2}{n} \le \frac{1}{n}\sum_{i=0}^{n-1} \tan^2(\frac{i\pi}{2n}) \le \frac{C_2}{n}.$$