Homework Assignment for Week 08

Assigned Nov 06, 2009. Due Nov 13, 2009.

1. Section 4.6: Problems 1, 2, 4, 11.

2. The solution to minimax problem is in general difficult, except for $n = 1$ (best linear approximation). Derive the formula of $m_1(x)$ for Example 4.4.1 on p160.

   Hint: If $f(-1) = f(1) = 0 = \max_{[-1,1]} f(x)$ and $f(0.3) = -2.1 = \min_{[-1,1]} f(x)$, then what is the best linear approximation $m_1(x)$ for $f(x)$ on $[-1, 1]$?

3. Does
$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \tan^2\left(\frac{i\pi}{2n}\right)$$
   converge? If so, what is the limit? If not, what is the leading order term in terms of n?

4. (Programming)

   Section 4.6: Do problem 6 for $g(x) = \frac{1}{1+x^2}$ on $-5 \leq x \leq 5$ with $n = 10$. Find the maximum of $|g(x) - p_{10}(x)|$ by sampling over $100 + 1$ uniformly spaced points. This is a clear contrast to Fig 4.6 on p143.

5. Challenge of the week with extra credit. Due Nov 20. Extension may be possible upon request, pending on your progress.

   Read section 4 of "On the maximum errors of polynomial ..."' by Powell. Ask me for hint on potential difficulties.

6. Another challenge of the weak with extra credit. Due time same as above.

   Find the best quadratic approximation $m_2(x)$ for e^x on $[-1, 1]$.

   This one may be very difficult for you. Be sure to talk to me first, before you spend time on it.