

Challenging Problems

1. Can you predict the order of accuracy of 'Natural Cubic Spline' applied to $\sin(\pi x)$ and $\cos(\pi x)$ on $[0, 1]$, respectively? Why?
2. Evaluate the improper integrals $\int_0^1 \frac{1}{\sin^{\frac{1}{3}} x} dx$ and $\int_0^1 \frac{1}{\sin(x^{\frac{1}{3}})} dx$ with fourth order accuracy.
3. The files f1.txt and f2.txt contain the data $(n, x_n, f_1(x_n), f_1'(x_n))$ and $(n, x_n, f_2(x_n), f_2'(x_n))$ with $x_n = 0, 1/256, 2/256, \dots, 1$, respectively. Both f_1 and f_2 has a singularity at $x = 0$. Can you identify the degrees of singularity and propose an accurate numerical differentiation for them? You could use the data with, for example, $h = 1/64, 1/128, 1/256$ to check the order of convergence of your approach.