HW4-4

(Look for p200. Theorem 4.5 and Theorem 4.6.)

(1)

Assume the expansion:

$$I = I_h + C_1 h^{P_1} + C_2 h^{P_2} + \cdots\cdots \quad , \text{where } 0 < P_1 < P_2 < \cdots\cdots$$

and $C_1, C_2 \cdots$ are constant.

when h small enough

$$I \approx I_h + C_1 h^{P_1} \qquad (\because P_1 < P_2 < \cdots\cdots)$$

If we know the exact integral I, then

$$\log_2\left(\frac{I - I_{2h}}{I - I_h}\right) \approx \log_2\left(\frac{\cancel{C_1}(2h)^{P_1}}{\cancel{C_1}(h)^{P_1}}\right) = \log_2(2^{P_1}) = P_1$$

Numerical result:

n	h	P_1 of midpoint	P_1 of trapezoidal
17	0.0625000	2.0019743	2.0002819
33	0.0312500	2.0004932	2.0000704
65	0.0156250	2.0001233	2.0000176
129	0.0078125	2.0000308	2.0000044
257	0.0039063	2.0000077	2.0000011

$\Rightarrow$ The rate of convergence of both two are $O(h^2)$ #

(2) If we don't know the exact Integral I

$$\text{Consider: } \log_2\left(\frac{I_{2h} - I_{4h}}{I_h - I_{2h}}\right) \approx \log_2\left(\frac{\cancel{C_1}((2h)^{P_1} - (4h)^{P_1})}{\cancel{C_1}((h)^{P_1} - (2h)^{P_1})}\right) = \log_2(2^{P_1}) = P_1$$