HW4-3

(a)

Let $P(x) = ax^2 + bx + c$

$$\begin{cases} P(-h) = ah^2 - bh + c = f(-h) \cdots\cdots ① \\ P(0) = \qquad\qquad c = f(0) \cdots\cdots ① \\ P(h) = ah^2 + bh + c = f(h) \cdots\cdots ③ \end{cases}$$

③ - ① $\Rightarrow$ $b = \dfrac{f(h) - f(-h)}{2h}$

② $\Rightarrow$ $c = f(0)$

substitute b, c into ①

$$\Rightarrow a = \frac{1}{h^2}\left(\frac{f(h)+f(-h)}{2} - f(0)\right)$$

$$\Rightarrow P(x) = \frac{1}{h^2}\left(\frac{f(h)+f(-h)}{2} - f(0)\right)x^2 + \frac{f(h)-f(-h)}{2h}x + f(0) \quad \#$$

(b)

$$\int_{-h}^{h} P(x) = \left(\frac{1}{3h^2}\left(\frac{f(h)-f(-h)}{2} - f(0)\right)x^3 + \frac{f(h)-f(-h)}{4h}x^2 + f(0)x\right)\Bigg|_{-h}^{h}$$

$$= 2h\left(\frac{f(-h)}{6} + \frac{4f(0)}{6} + \frac{f(h)}{6}\right) \quad \#$$

(c) Let $y = Cx + D$

$$\begin{cases} C(-h) + D = a \\ C(h) + D = b \end{cases} \Rightarrow \begin{cases} C = \dfrac{b-a}{2h} \\ D = \dfrac{a+b}{2} \end{cases}$$

$$\int_a^b f(y)\,dy = \int_{-h}^{h} f(Cx+D)\cdot C\,dx = C\cdot 2h\left(\frac{f(C\cdot(-h)+D)}{6} + \frac{4f(D)}{6} + \frac{f(Ch+D)}{6}\right)$$

$$= \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right) \quad \#$$