

HW3 - section 4.2 - 12 - extra.

Sol: Assuming the expansion:

$$e = \left(\frac{2+h}{2-h}\right)^{\frac{1}{n}} + C_1 h^{P_1} + C_2 h^{P_2} + \dots$$

$$\therefore \left(\frac{2+h}{2-h}\right)^{\frac{1}{n}} \rightarrow e \text{ as } h \rightarrow 0$$

$$\therefore P_1 > 0, \text{ and } P_1 < P_2 < P_3 \dots$$

$$\text{Let } N(h) = \left(\frac{2+h}{2-h}\right)^{\frac{1}{n}}$$

$$= e - C_1 h^{P_1} - C_2 h^{P_2} - \dots$$

$$\approx e - C_1 h^{P_1} \text{ when } h \text{ small enough}$$

$$\text{Consider: } \frac{N(h) - N(\frac{h}{2})}{N(\frac{h}{2}) - N(\frac{h}{4})} \approx \frac{C_1[(\frac{h}{2})^{P_1} - (h)^{P_1}]}{C_1[(\frac{h}{4})^{P_1} - (\frac{h}{2})^{P_1}]} \\ = 2^{P_1}$$

$$\Rightarrow \log_2 \left(\frac{N(h) - N(\frac{h}{2})}{N(\frac{h}{2}) - N(\frac{h}{4})} \right) \approx P_1$$

$$\text{數值結果: } n=10, P_1 = 2.0025956$$

$$n=100, P_1 = 2.000259$$

$$n=200, P_1 = 2.000070$$

$$n=400, P_1 = 2.000018 \Rightarrow P_1 \approx 2$$

$\Rightarrow K_1 = 0$ (因為最低階的 P_1 是 2, 不是 1) #

