5.

$$[A|b] = \left[\begin{array}{cccccccccc|c}
a_{11} & a_{12} & 0 & \cdots & 0 & a_{1,N+1} & 0 & \cdots & \cdots & 0 & b_1 \\
a_{21} & a_{22} & a_{23} & 0 & \cdots & 0 & a_{2,N+2} & & & & b_2 \\
0 & a_{32} & a_{33} & a_{34} & & & & a_{3,N+3} & & 0 & b_3 \\
\vdots & & \ddots & \ddots & \ddots & & & & \ddots & & \vdots \\
0 & & & & & & & & & 0 & \vdots \\
a_{N+1,1} & 0 & & & & & & & & a_{N^2-N,N^2} & \vdots \\
0 & a_{N+2,1} & 0 & & & & & & & 0 & \vdots \\
& 0 & a_{N+3,1} & & \ddots & & & & & \vdots & \vdots \\
\vdots & & & \ddots & & & \ddots & & & 0 & \vdots \\
& & 0 & & & & & & & a_{N^2-1,N^2} & b_{N^2-1} \\
0 & \cdots & \cdots & 0 & a_{N^2,N^2-N} & 0 & \cdots & 0 & a_{N^2,N^2-1} & a_{N^2N^2} & b_{N^2}
\end{array}\right] = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ \vdots \\ E_{N^2} \end{bmatrix}$$

$$= [A^{(1)} | b^{(1)}] = \begin{bmatrix} E_1^{(1)} \\ \vdots \\ E_{N^2}^{(1)} \end{bmatrix}$$

1

Use $E_1^{(1)}$ to eliminate x_1 from the equation $E_2^{(1)}$, $E_{N+1}^{(1)}$

we need $2+3+3 = 2+2\times 3 = 2(3+1)$ multip.

and $(E_2^{(1)} - m_{12}E_1^{(1)}) \rightarrow E_2^{(2)}$, $(E_{N+1}^{(1)} - m_{1,N+1}E_1^{(1)}) \rightarrow E_{N+1}^{(2)}$

$\Rightarrow$ $E_2^{(2)}(N+1) \neq 0$, $E_{N+1}^{(2)}(2) \neq 0$

Use $E_2^{(2)}$ to eliminate x_2 from the equation $E_3^{(2)}$, $\underline{E_{N+1}^{(2)}}$, $E_{N+2}^{(2)}$

$\because$ the row 2 of $A^{(2)}$ has 4 nonzero entries

$\therefore$ we need $3+3\times 4 = 3(4+1)$ multip.

$\vdots$

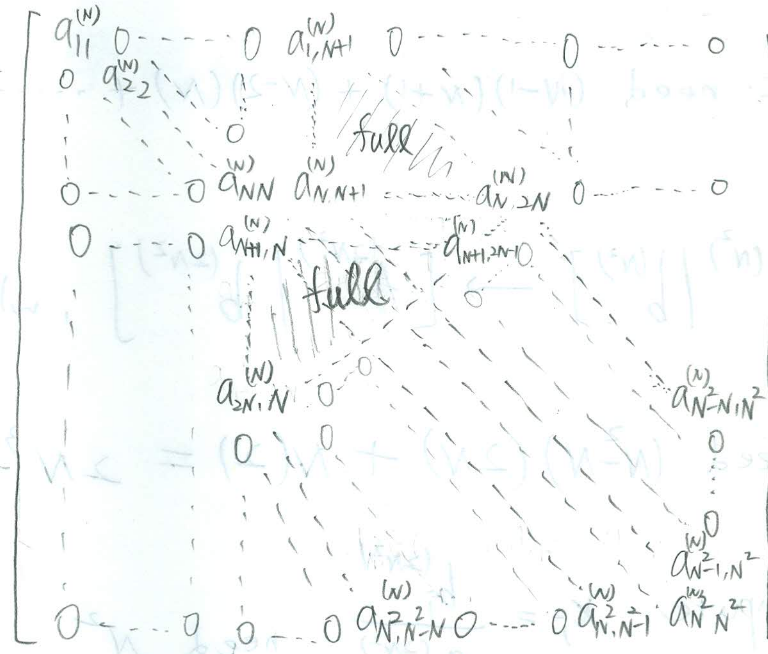
use $E_{N-1}^{(N-1)}$ to eliminate x_{N-1} from the equation $E_N^{(N-1)}, E_{N+1}^{(N-1)}, \cdots, E_{2N-1}^{(N-1)}$

$\because$ the row $N-1$ of $A^{(N-1)}$ has N nonzero entries

$\therefore$ need $N-1+(N-1)N = (N-1)(N+1)$

③

Now, $\left[A^{(N)} \mid b^{(N)}\right] =$



$\because$ the row N of $A^{(N)}$ has $N+1$ nonzero entries

and use $E_N^{(N)}$ to eliminate x_N from the equation

$\underbrace{E_{N+1}^{(N)}, E_{N+2}^{(N)}, \cdots, E_{2N}^{(N)}}_{N}$ we need $N+(N+1)\cdot N$

$= (N+2)N$

until, use $E_{N^2-N}^{(N^2-N)}$ to eliminate x_{N^2-N} from the equation

$\underbrace{E_{N^2-N+1}^{(N^2-N)}, E_{N^2-N+2}^{(N^2-N)}, \cdots\cdots E_{N^2}^{(N)}}_{N}$, we need $N+(N+1)\cdot N$

$= (N+2)N$

③ use $E_{N^2-N+1}^{(N^2-N+1)}$ to eliminate x_{N^2-N+1} from the equation

$$\underbrace{E_{N^2-N+2}^{(N^2-N+1)},\ E_{N^2-N+3}^{(N^2-N+1)},\ \cdots\cdots,\ E_{N^2}^{(N^2-N+1)}}_{\text{need } N-1+(N-1)N = (N-1)(N+1)}$$

untill $\rightarrow \left[A^{(N^2)} \mid b^{(N^2)}\right]$, where $A^{(N^2)}$ is upper triangular matrix

It need $(N-1)(N+1) + (N-2)(N) + \cdots\cdots + (1)(3)$

④ $\left[A^{(N^2)} \mid b^{(N^2)}\right] \longrightarrow \left[A^{(2N^2)} \mid b^{(2N^2)}\right]$, where $A^{(2N^2)}$ is diagonal matrix

$\Rightarrow$ we need $(N^2-N)(2N) + N(2) = 2N^3 - 2N^2 + 2N$

⑤ compute $x_i = \dfrac{b_i^{(2N^2)}}{a_{ii}^{(2N^2)}}$ need N^2

By ①②③④⑤ total: $2\times4 + 3\times5 + \cdots + (N-1)(N+1)$

$+ (N^2-2N+1)(N)(N+2)$

$+ (N-1)(N+1) + (N-2)(N) + \cdots + (1)(3)$

$+ 2N^3 - 2N^2 + 2N + N^2$

$= \underline{N^4} + \cdots\cdots\cdots$

※