3.

$$\because \quad [A|b] = \left[\begin{array}{ccccccccc|c} a_{11} & a_{12} & a_{13} & 0 & \cdots & & & & 0 & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & \cdots & & & 0 & b_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & 0 & \cdots & & 0 & b_3 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0 & \cdots & 0 & b_4 \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & & \vdots & \vdots \\ & & & & & & & & 0 & \\ & & & & & & & & a_{N^2-2,N^2} & \\ & & & & & & & & a_{N^2-1,N^2} & \\ 0 & \cdots & & & 0 & a_{N^2,N^2-2} & & a_{N^2,N^2-1} & a_{N^2,N^2} & b_{N^2} \end{array}\right] = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_{N^2} \end{bmatrix} = \begin{bmatrix} E_1^{(1)} \\ E_2^{(1)} \\ \vdots \\ E_{N^2}^{(1)} \end{bmatrix}$$

$$[A|b] = [A^{(1)}|b^{(1)}]$$

① Use $E_1^{(1)}$ to eliminate x_1 from the equation $E_2^{(1)}, E_3^{(1)}$,

we need compute $m_{12} = \frac{a_{21}^{(1)}}{a_{11}^{(1)}}$ need 1 multip.

$m_{13} = \frac{a_{31}^{(1)}}{a_{11}^{(1)}}$ need 1 multip.

and $(E_2^{(1)} - m_{12}E_1^{(1)}) \to E_2^{(2)}$ need 3 multip.

$(E_3^{(1)} - m_{13}E_1^{(1)}) \to E_3^{(2)}$ need 3 multip.

So, total unmber of multiplications is $2+3+3=8$

$\Rightarrow$ Use $E_i^{(i)}$ to eliminate x_i from the equation $E_{i+1}^{(i)}, E_{i+2}^{(i)}$,

we need compute $m_{i,i+1}$ need 1 multip.

$m_{i,i+2}$ need 1 multip.

and $(E_{i+1}^{(i)} - m_{i,i+1}E_i^{(i)}) \to E_{(i+1)}^{(i+1)}$ need 3 multip.

$(E_{i+2}^{(i)} - m_{i,i+2}E_i^{(i)}) \to E_{(i+2)}^{(i+1)}$ need 3 multip.

for $i = 1, 2, \ldots\ldots N^2-2$.

& for $i = N^2-1$:

we need compute $m_{N^2-1,N^2} = \frac{a^{(N^2-1)}_{N^2,N^2-1}}{a^{(N^2-1)}_{N^2-1,N^2-1}}$ need 1 multip.

and $(E_{N^2}^{(N^2-1)} - m_{N^2-1,N^2}E_{N^2-1}^{(N^2-1)}) \to E_{N^2}^{(N^2)}$ we need 2 multip.

So,

$$[A \mid b] \rightarrow \left[\begin{array}{cccccc|c} a_{11}^{(N^2)} & a_{12}^{(N^2)} & a_{13}^{(N^2)} & 0 & \cdots & 0 & b_1^{(N^2)} \\ & \ddots & \ddots & \ddots & & \vdots & b_2^{(N^2)} \\ & & \ddots & & \ddots & 0 & \vdots \\ & & & \ddots & & a_{N^2-2,N^2}^{(N^2)} & \vdots \\ & & & & \ddots & a_{N^2-1,N^2}^{(N^2)} & \vdots \\ & & & & & a_{N^2N^2}^{(N^2)} & b_{N^2}^{(N^2)} \end{array}\right] = [A^{(N^2)} \mid b^{(N^2)}]$$

we need $(N^2-2)\times 8 + 3 = 8N^2-13$

② Use $E_{N^2}^{(N^2)}$ to eliminate x_{N^2} from the equation $E_{N^2-1}^{(N^2)}, E_{N^2-2}^{(N^2)}$

we need compute m_{N^2,N^2-1} need 1 multip.

m_{N^2,N^2-2} need 1 multip.

and $(E_{N^2-1}^{(N^2)} - m_{N^2,N^2-1}E_{N^2}^{(N^2)}) \rightarrow E_{N^2-1}^{(N^2+1)}$ need 1 multip.

$(E_{N^2-2}^{(N^2)} - m_{N^2,N^2-2}E_{N^2}^{(N^2)}) \rightarrow E_{N^2-2}^{(N^2+1)}$ need 1 multip.

$$\Rightarrow \quad [A^{(N^2)} \mid b^{(N^2)}] \rightarrow \left[\begin{array}{ccc|c} a_{11}^{(2N^2)} & & 0 & b_1^{(2N^2)} \\ & \ddots & & \vdots \\ 0 & & a_{N^2N^2}^{(2N^2)} & b_{N^2}^{(2N^2)} \end{array}\right]$$

we need $(N^2-1)(4)+2 = 4N^2-6$

③

compute $x_i = \dfrac{b_i^{(2N^2)}}{a_{ii}^{(2N^2)}}$, $1 \le i \le N^2$

we need N^2 multip.

By ①②③ we need $8N^2-13+4N^2-6+N^2 = 13N^2-19$

♯