HW 07

2. Let $h = \frac{1}{N}$, $x_j = jh = \frac{j}{N}$, $u(x_j) = u_j$, for $j = 0, 1, 2, \cdots, N$

$$u''(x_j) = \frac{u_{j-1} - 2u_j + u_{j+1}}{h^2}, \quad \text{for } j = 1, 2, \cdots, N-1$$

$$u'(x_j) = \frac{u_{j+1} - u_{j-1}}{2h}, \quad \text{for } j = 1, 2, \cdots, N-1$$

$$\Rightarrow \underbrace{\frac{1}{h^2}\begin{bmatrix} -2 & 1 & 0 & 0 & \cdots & & 0 \\ 1 & -2 & 1 & 0 & & & \vdots \\ 0 & 1 & -2 & 1 & & & \\ \vdots & & \ddots & \ddots & \ddots & & \\ & & & 1 & -2 & 1 & 0 \\ & & & 0 & 1 & -2 & 1 \\ 0 & \cdots & & 0 & 0 & 1 & -2 \end{bmatrix}}_{\text{Let } = A} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix}}_{= \vec{u}} + \begin{bmatrix} u_0/h^2 = \alpha/h^2 \\ 0 \\ \vdots \\ 0 \\ -u_N/h^2 = -\beta/h^2 \end{bmatrix} = \begin{bmatrix} u''(x_1) \\ u''(x_2) \\ \vdots \\ u''(x_{N-1}) \end{bmatrix}$$

$$\underbrace{\frac{1}{2h}\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & & 0 \\ -1 & 0 & 1 & 0 & & & \vdots \\ 0 & -1 & 0 & 1 & & & \\ \vdots & & \ddots & \ddots & \ddots & & \\ & & & -1 & 0 & 1 & 0 \\ & & & 0 & -1 & 0 & 1 \\ 0 & \cdots & & 0 & 0 & -1 & 0 \end{bmatrix}}_{\text{Let } = B} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{bmatrix}}_{= \vec{u}} + \begin{bmatrix} -u_0/2h = -\alpha/2h \\ 0 \\ \vdots \\ 0 \\ u_N/2h = \beta/2h \end{bmatrix} = \begin{bmatrix} u'(x_1) \\ u'(x_2) \\ \vdots \\ u'(x_{N-1}) \end{bmatrix}$$

$$\Rightarrow (A+B)\vec{u} + \begin{bmatrix} \alpha(\frac{1}{h^2} - \frac{1}{2h}) \\ 0 \\ \vdots \\ 0 \\ \beta(\frac{1}{2h} - \frac{1}{h^2}) \end{bmatrix} = \begin{bmatrix} u''(x_1) + u'(x_1) \\ u''(x_2) + u'(x_2) \\ \vdots \\ u''(x_{N-1}) + u'(x_{N-1}) \end{bmatrix} = \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_{N-1}) \end{bmatrix}$$