1. look Chap 06 slides (new) P.73

But if A is singular

ex: $A = \begin{pmatrix} 3 & 2 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

then $L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & * & 1 \end{pmatrix}$, $U = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow LU = A$, where $*$ be arbitrary

so, the LU factorization is not unique.

#

2. look chap06 slides (new) P75 ~ P76

Corollary: The matrix A is positive definite if and only if

$A = GG^T$, where G is lower triangular with positive diagonal entries

& The algorithm (Cholesky Factorization) look P79

#

3. try to compute GG^t

Let $G = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ 0 & g_{32} & g_{33} & 0 \\ 0 & 0 & g_{43} & g_{44} \end{pmatrix}$

$$GG^t = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ 0 & g_{32} & g_{33} & 0 \\ 0 & 0 & g_{43} & g_{44} \end{pmatrix} \begin{pmatrix} g_{11} & g_{21} & 0 & 0 \\ 0 & g_{22} & g_{32} & 0 \\ 0 & 0 & g_{33} & g_{43} \\ 0 & 0 & 0 & g_{44} \end{pmatrix}$$

$$= \begin{pmatrix} g_{11}^2 & g_{11}\cdot g_{21} & 0 & 0 \\ g_{11}\cdot g_{21} & g_{21}^2+g_{22}^2 & g_{22}\cdot g_{32} & 0 \\ 0 & g_{22}\cdot g_{32} & g_{32}^2+g_{33}^2 & g_{33}\cdot g_{43} \\ 0 & 0 & g_{33}\cdot g_{43} & g_{43}^2+g_{44}^2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}$$

$\Rightarrow g_{11}^2 = a_{11} \Rightarrow g_{11} = a_{11}^{\frac{1}{2}}$;

$g_{21} = a_{21}/g_{11}$;

for $i = 2 : n-1$

$\quad g_{ii} = (a_{ii} - g_{i,i-1}^2)^{1/2}$

$\quad g_{i+1,i} = a_{i+1,i}/g_{ii}$

end

$g_{nn} = (a_{nn} - g_{n,n-1}^2)^{1/2}$

need "n" sqrt

$\Rightarrow$ 10n multip.

need $1 + \underbrace{2+2+\cdots+2}_{n-2} + 1$

$\Rightarrow$ 2n-2 multip.

so total need 12n-2 multip.

$\Rightarrow C=12,\ p=1$ ※

4.

$$A = \begin{bmatrix} 5 & 2 & & & 0 \\ 2 & 5 & 2 & & \\ & \ddots & \ddots & \ddots & \\ & & 2 & 5 & 2 \\ 0 & & & 2 & 5 \end{bmatrix}_{20\times 20}$$

$\because$ $a_{ii} = 5 \geq 4 \geq \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|$ for each $i = 1, 2, \cdots, n$

$\therefore$ A is strictly diagonally dominant

By Thm 6.19 Do G-E without row interchanges.

& each of the matrices $A^{(2)}, A^{(3)}, \cdots, A^{(n)}$ generated by the Gaussian elimination process is strictly diagonally dominant

for G-E with partial pivoting

$\because$ $a_{11} = 5 > 2 = a_{21}$ $\therefore$ the first step without row interchanges.

and $\because$ each $A^{(i)}$ is strictly diagonally dominant

$\therefore$

$$A^{(i)} = \begin{pmatrix} a^{(i)}_{i,i} & a^{(i)}_{i,i+1} & \\ a^{(i)}_{i+1,i} & a^{(i)}_{i+1,i+1} & a^{(i)}_{i+1,i+2} \\ & & \ddots \end{pmatrix}$$

$a^{(i)}_{i,i} > a^{(i)}_{i,i+1} = a^{(i)}_{i+1,i} = 2$ ($\because$ A是三對角 $\therefore$ $a^{(i)}_{i,i+1} = a_{i,i+1}$ 不變)

$\Rightarrow$ Do G-E with partial pivoting without any row interchange. ※

5.

```
b = ones(20,1) ;
A = diag( 5.1 * ones(20,1) )
  + diag ( 2 * ones (19,1), 1 )
  + diag (2 * ones( 19,1), -1) ;

[L U] = lu(A) ;

z = zeros(20,1);
z(20) = 1/U(20,20) ;

for i = 19 : -1 : 1
    z(i) = ( b(i) - U(i,i+1) * z(i+1) ) / U(i,i)
end
```

so,

<u>error</u> = max (abs (U*z - b))

$= 1.1102 \times 10^{-16}$.

#