Week 09

2. ① if $A = LU$ (without row interchanges),

with L unit lower triangular and U upper triangular.

then $A_k = L_k U_k$, A_k is the $k \times k$ leading principle of A

L_k 〃 〃 〃 〃 〃 〃 L

U_k 〃 〃 〃 〃 〃 〃 U

②

$$\det(A_k) = \det(L_k U_k) = \det(L_k) \times \det(U_k)$$
$$= 1 \times \det(U_k) = \det(U_k)$$

③

proof of Theorem 6.24:

By Theorem 6.23 we have " A is s.p.D. matrix $\Leftrightarrow$ $\det(A_k) > 0$ for $1 \le k \le n$

$\overset{\text{by ②}}{\Leftrightarrow}$ $\det(U_k) > 0$ for $1 \le k \le n$

$\Leftrightarrow$ $u_{11} > 0$, $u_{22} > 0$, ..., $u_{nn} > 0$

$\Leftrightarrow$ Gaussian elimination without row interchanges can be performed on the linear system $Ax = b$ with all pivot elements positive

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