which results in the linear equations

$$u(x_i) = x_i^2 + \frac{1}{12}\left[e^{x_i}u(0) + 4e^{|x_i - \frac{1}{4}|}u\left(\frac{1}{4}\right) + 2e^{|x_i - \frac{1}{2}|}u\left(\frac{1}{2}\right) + 4e^{|x_i - \frac{3}{4}|}u\left(\frac{3}{4}\right) + e^{|x_i - 1|}u(1)\right].$$

The 5×5 linear system has solutions $u(0) = -1.234286$, $u\left(\frac{1}{4}\right) = -0.9507292$, $u\left(\frac{1}{2}\right) = -0.7659400$, $u\left(\frac{3}{4}\right) = -0.5844737$, and $u(1) = -0.4484975$.

Exercise Set 6.2, page 368

1. The following row interchanges are required for these systems.

   (a) none

   (b) Interchange rows 2 and 3.

   (c) none

   (d) Interchange rows 1 and 2.

2. The following row interchanges are required for these systems.

   (a) none

   (b) none

   (c) none

   (d) none

3. The following row interchanges are required for these systems.

   (a) Interchange rows 1 and 2.

   (b) Interchange rows 1 and 3.

   (c) Interchange rows 1 and 2, then interchange rows 2 and 3.

   (d) Interchange rows 1 and 2.

4. The following row interchanges are required for these systems.

   (a) Interchange rows 2 and 3.

   (b) Interchange rows 1 and 3.

   (c) Interchange rows 1 and 3, then interchange rows 2 and 3.

   (d) Interchange rows 1 and 2.

5. The following row interchanges are required for these systems.

   (a) Interchange rows 1 and 3, then interchange rows 2 and 3.

   (b) Interchange rows 2 and 3.

   (c) Interchange rows 2 and 3.

   (d) Interchange rows 1 and 3, then interchange rows 2 and 3.

6. The following row interchanges are required for these systems.

   (a) Interchange rows 2 and 3. (b) none

   (c) Interchange rows 1 and 2, then interchange rows 2 and 3.

   (d) none

7. The following row interchanges are required for these systems.

   (a) Interchange rows 1 and 2, and columns 1 and 3, then interchange rows 2 and 3, and columns 2 and 3.

   (b) Interchange rows 1 and 2, and columns 1 and 3, then interchange rows 2 and 3.

   (c) Interchange rows 1 and 2, and columns 1 and 3, then interchange rows 2 and 3.

   (d) Interchange rows 1 and 2, and columns 1 and 2, then interchange rows 2 and 3; and columns 2 and 3.

8. The following row interchanges are required for these systems.

   (a) Interchange rows 1 and 2, and columns 1 and 3.

   (b) Interchange rows 1 and 2, and columns 1 and 2, then interchange rows 2 and 3.

   (c) Interchange rows 1 and 3, and columns 1 and 2, then interchange rows 2 and 3, and columns 2 and 3.

   (d) Interchange rows 1 and 2.

9. Gaussian elimination with three-digit chopping arithmetic gives the following results.

   (a) $x_1 = 30.0, x_2 = 0.990$ (b) $x_1 = 0.00, x_2 = 10.0, x_3 = 0.142$

   (c) $x_1 = 0.206, x_2 = 0.0154, x_3 = -0.0156, x_4 = -0.716$

   (d) $x_1 = 0.828, x_2 = -3.32, x_3 = 0.153, x_4 = 4.91$

10. Gaussian elimination with three-digit chopping arithmetic gives the following results.

    (a) $x_1 = 1.00,\ x_2 = 9.98$ (b) $x_1 = 12.0,\ x_2 = 0.492, x_3 = -9.78$

    (c) $x_1 = -8.25, x_2 = -8.00, x_3 = -0.0339, x_4 = 0.0566$

    (d) $x_1 = 1.33, x_2 = -4.66, x_3 = -4.04, x_4 = -1.66$

11. Gaussian elimination with three-digit rounding arithmetic gives the following results.

    (a) $x_1 = -10.0,\ x_2 = 1.01$ (b) $x_1 = 0.00,\ x_2 = 10.0,\ x_3 = 0.143$

    (c) $x_1 = 0.185,\ x_2 = 0.0103,\ x_3 = -0.0200,\ x_4 = -1.12$

    (d) $x_1 = 0.799,\ x_2 = -3.12,\ x_3 = 0.151,\ x_4 = 4.56$

12. Gaussian elimination with three-digit rounding arithmetic gives the following results.

(a) $x_1 = 1.00$, $x_2 = 10.0$ (b) $x_1 = 12.0$, $x_2 = 0.499$, $x_3 = -1.98$

(c) $x_1 = 0.0896, x_2 = -0.0639, x_3 = -0.0361, x_4 = 0.0467$

(d) $x_1 = 1.35, x_2 = -4.73, x_3 = -4.07, x_4 = -1.65$

13. Gaussian elimination with partial pivoting and three-digit chopping arithmetic gives the following results.

(a) $x_1 = 10.0$, $x_2 = 1.00$ (b) $x_1 = -0.163$, $x_2 = 9.98, x_3 = 0.142$

(c) $x_1 = 0.177$, $x_2 = -0.0072$, $x_3 = -0.0208$, $x_4 = -1.18$

(d) $x_1 = 0.777$, $x_2 = -3.10$, $x_3 = 0.161$, $x_4 = 4.50$

14. Gaussian elimination with partial pivoting gives the following results.

(a) $x_1 = 1.00$, $x_2 = 9.98$ (b) $x_1 = 12.0$, $x_2 = 0.504$, $x_3 = -9.78$

(c) $x_1 = 0.0928, x_2 = -0.0631, x_3 = -0.0356, x_4 = 0.0468$

(d) $x_1 = 1.33, x_2 = -4.66, x_3 = -4.04, x_4 = -1.66$

15. Gaussian elimination with partial pivoting and three-digit rounding arithmetic gives the following results.

(a) $x_1 = 10.0$, $x_2 = 1.00$ (b) $x_1 = 0.00$, $x_2 = 10.0$, $x_3 = 0.143$

(c) $x_1 = 0.178$, $x_2 = 0.0127$, $x_3 = -0.0204$, $x_4 = -1.16$

(d) $x_1 = 0.845$, $x_2 = -3.37$, $x_3 = 0.182$, $x_4 = 5.07$

16. Gaussian elimination with partial pivoting and three-digit chopping arithmetic gives the following results.

(a) $x_1 = 1.00$, $x_2 = 10.0$ (b) $x_1 = 12.0$, $x_2 = 0.499$, $x_3 = -1.98$

(c) $x_1 = 0.0927, x_2 = -0.0631, x_3 = -0.0362, x_4 = 0.0465$

(d) $x_1 = 1.35, x_2 = -4.73, x_3 = -4.07, x_4 = -1.65$

17. Gaussian elimination with scaled partial pivoting and three-digit chopping arithmetic gives the following results.

(a) $x_1 = 10.0$, $x_2 = 1.00$ (b) $x_1 = -0.163$, $x_2 = 9.98$, $x_3 = 0.142$

(c) $x_1 = 0.171$, $x_2 = 0.0102$, $x_3 = -0.0217$, $x_4 = -1.27$

(d) $x_1 = 0.687$, $x_2 = -2.66$, $x_3 = 0.117$, $x_4 = 3.59$

18. Gaussian elimination with scaled partial pivoting gives the following results.

(a) $x_1 = 1.00$, $x_2 = 9.98$ (b) $x_1 = 0.993$, $x_2 = 0.500$, $x_3 = -1.00$

(c) $x_1 = 0.0930, x_2 = -0.0631, x_3 = -0.0359, x_4 = 0.0467$

(d) $x - 1 = 1.33, x_2 = -4.66, x_3 = -4.04, x_4 = -1.66$

19. Gaussian elimination with scaled partial pivoting and three-digit rounding arithmetic gives the following results.

(a) $x_1 = 10.0$, $x_2 = 1.00$ (b) $x_1 = 0.00$, $x_2 = 10.0$, $x_3 = 0.143$

(c) $x_1 = 0.180$, $x_2 = 0.0128$, $x_3 = -0.0200$, $x_4 = -1.13$

(d) $x_1 = 0.783$, $x_2 = -3.12$, $x_3 = 0.147$, $x_4 = 4.53$

20. Gaussian elimination with scaled partial pivoting and three-digit chopping arithmetic gives the following results.

(a) $x_1 = 1.00$, $x_2 = 10.0$ (b) $x_1 = 1.03$, $x_2 = 0.502$, $x_3 = -1.01$

(c) $x_1 = 0.0927, x_2 = -0.0630, x_3 = -0.0360, x_4 = 0.0467$

(d) $x_1 = 1.35, x_2 = -4.73, x_3 = -4.07, x_4 = -1.65$

21. Using Algorithm 6.1 in Maple with `Digits:=10` gives

(a) $x_1 = 10.00000000, x_2 = 1.000000000$

(b) $x_1 = 0.000000033, x_2 = 10.00000001, x_3 = 0.1428571429$

(c) $x_1 = 0.1768252958, x_2 = 0.0126926913, x_3 = -0.0206540503, x_4 = -1.182608714$

(d) $x_1 = 0.7883937842, x_2 = -3.125413672, x_3 = 0.1675965951, x_4 = 4.557002521$

22. Using Algorithm 6.1 in Maple with `Digits:=10` gives

(a) $x_1 = 1.000000000, x_2 = 10.000000000$

(b) $x_1 = 1.000000300, x_2 = 0.500000001, x_3 = -1.000000306$

(c) $x_1 = 0.0927610467, x_2 = -0.06299433926, x_3 = -0.03624582267, x_4 = 0.04670801939$

(d) $x_1 = 1.349448559, x_2 = -4.677987755, x_3 = -4.032893779, x_4 = -1.656637732$

23. Using Algorithm 6.2 in Maple with `Digits:=10` gives

(a) $x_1 = 10.00000000, x_2 = 1.000000000$

(b) $x_1 = 0.000000000, x_2 = 10.00000000, x_3 = 0.142857142$

(c) $x_1 = 0.1768252975, x_2 = 0.0126926909, x_3 = -0.0206540502, x_4 = -1.182608696$

(d) $x_1 = 0.7883937863, x_2 = -3.125413680, x_3 = 0.1675965980, x_4 = 4.557002510$

24. Using Algorithm 6.2 in Maple with `Digits:=10` gives

(a) $x_1 = 1.000000000, x_2 = 10.000000000$

(b) $x_1 = 1.000000300, x_2 = 0.500000001, x_3 = -1.000000306$

(c) $x_1 = 0.09276104704, x_2 = -0.06299433961, x_3 = -0.03624582264, x_4 = 0.04670801938$

(d) $x_1 = 1.349448559, x_2 = -4.677987755, x_3 = -4.032893779, x_4 = -1.656637732$

25. Using Algorithm 6.3 in Maple with `Digits:=10` gives

(a) $x_1 = 10.00000000, x_2 = 1.000000000$

(b) $x_1 = 0.000000000, x_2 = 10.00000000, x_3 = 0.1428571429$

(c) $x_1 = 0.1768252977, x_2 = 0.0126926909, x_3 = -0.0206540501, x_4 = -1.182608693$

(d) $x_1 = 0.7883937842, x_2 = -3.125413672, x_3 = 0.1675965952, x_4 = 4055700252$

26. Using Algorithm 6.3 in Maple with `Digits:=10` gives

(a) $x_1 = 1.000000000, x_2 = 10.000000000$

(b) $x_1 = 1.000000000, x_2 = 0.500000000, x_3 = -1.000000000$

(c) $x_1 = 0.09276104705, x_2 = -0.06299433961, x_3 = -0.03624582264, x_4 = 0.04670801938$

(d) $x_1 = 1.349448559, x_2 = -4.677987755, x_3 = -4.032893779, x_4 = -1.656637732$

27. Using Gaussian elimination with complete pivoting gives:

(a) $x_1 = 9.98, x_2 = 1.00$

(b) $x_1 = 0.0724, x_2 = 10.0, x_3 = 0.0952$

(c) $x_1 = 0.161, x_2 = 0.0125, x_3 = -0.0232, x_4 = -1.42$

(d) $x_1 = 0.719, x_2 = -2.86, x_3 = 0.146, x_4 = 4.00$

28. Gaussian elimination with complete pivoting gives the following results.

(a) $x_1 = 1.00, x_2 = 9.98$

(b) $x_1 = 0.982, x_2 = 0.500, x_3 = -0.994$

(c) $x_1 = 0.0933, x_2 = -0.0631, x_3 = -0.0360, 0.0464$

(d) $x_1 = 1.33, x_2 = -4.66, x_3 = -4.04, x_4 = -1.65$

29. Using Gaussian elimination with complete pivoting and three-digit rounding arithmetic gives:

(a) $x_1 = 10.0, x_2 = 1.00$

(b) $x_1 = 0.00, x_2 = 10.0, x_3 = 0.143$

(c) $x_1 = 0.179, x_2 = 0.0127, x_3 = -0.0203, x_4 = -1.15$

(d) $x_1 = 0.874, x_2 = -3.49, x_3 = 0.192, x_4 = 5.33$

30. Gaussian elimination with complete pivoting and three-digit rounding gives the following results.

(a) $x_1 = 10.0$, $x_2 = 1.00$ (b) $x_1 = 10.0$, $x_2 = 1.00$

(c) $x_1 = 0.0926, x_2 = -0.0629, x_3 = -0.0361, x_4 = 0.0466$

(d) $x_1 = 1.33, x_2 = -4.68, x_3 = -4.06, x_4 = -1.65$

31. The only system which does not require row interchanges is (a), where $\alpha = 6$.

32. Change Algorithm 6.2 as follows:

Add to STEP 1.

$$NCOL(i) = i$$

Replace STEP 3 with the following.

Let p and q be the smallest integers with $i \le p$, $q \le n$ and

$$|a(NROW(p), NCOL(q))| = \max_{i \le k, j \le n} |a(NROW(k), NCOL(j))|.$$

Add to STEP 4.

$$A(NROW(p), NCOL(q)) = 0$$

Add to STEP 5.

If $NCOL(q) \neq NCOL(i)$ then set

$$\begin{aligned} NCOPY &= NCOL(i); \\ NCOL(i) &= NCOL(q); \\ NCOL(q) &= NCOPY. \end{aligned}$$

Replace STEP 7 with the following.

Set

$$m(NROW(j), NCOL(i)) = \frac{a(NROW(j), NCOL(i))}{a(NROW(i), NCOL(i))}.$$

Replace in STEP 8:

$$m(NROW(j), i) \text{ by } m(NROW(j), NCOL(i))$$

Replace in STEP 9:

$$a(NROW(n), n) \text{ by } a(NROW(n), NCOL(n))$$

Replace STEP 10 with the following.

Set

$$X(NCOL(n)) = \frac{a(NROW(n), n+1)}{a(NROW(n), NCOL(n))}.$$

Replace STEP 11 with the following.

Set

$$X(NCOL(i)) = \frac{a(NROW(i), n+1) - \sum_{j=i+1}^{n} a(NROW(i), NCOL(j)) \cdot X(NCOL(j))}{A(NROW(i), NCOL(i))}.$$

Replace STEP 12 with the following.

$$\text{OUTPUT } (`X(', NCOL(i), `) =', X(NCOL(i)) \text{ for } i = 1, \ldots, n).$$

33. Using the Complete Pivoting Algorithm in Maple with Digits:=10 gives

    (a) $x_1 = 10.00000000, x_2 = 1.000000000$

    (b) $x_1 = 0.000000000, x_2 = 10.00000000, x_3 = 0.1428571429$

    (c) $x_1 = 0.1768252974, x_2 = 0.01269269087, x_3 = -0.02065405015, x_4 = -1.182608697$

    (d) $x_1 = 0.17883937840, x_2 = -3.125413669, x_3 = 0.1675965971, x_4 = 4.557002516$

34. Using the Complete Pivoting Algorithm in Maple with Digits:=10 gives

    (a) $x_1 = 1.000000000, x_2 = 10.000000000$

    (b) $x_1 = 1.000000001, x_2 = 0.5000000000, x_3 = -1.000000001$

    (c) $x_1 = 0.09276104701, x_2 = -0.06299433960, x_3 = -0.03624582267, x_4 = 0.04670801937$

    (d) $x_1 = 1.349448557, x_2 = -4.677987750, x_3 = -4.032893778, x_4 = -1.656637732$

Exercise Set 6.3, page 378

1. Determine if the matrices are nonsingular, and if so, find the inverse.

    (a) The matrix is singular.

    (b) $\begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{5}{8} & -\frac{1}{8} & -\frac{1}{8} \\ \frac{1}{8} & -\frac{5}{8} & \frac{3}{8} \end{bmatrix}$

    (c) The matrix is singular.

    (d) $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ -\frac{3}{14} & \frac{1}{7} & 0 & 0 \\ \frac{3}{28} & -\frac{11}{7} & 1 & 0 \\ -\frac{1}{2} & 1 & -1 & 1 \end{bmatrix}$

2. Determine if the matrices are nonsingular, and if so, find the inverse.

    (a)

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ -1 & 4 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \\ -\frac{1}{10} & \frac{3}{5} & -\frac{1}{10} \end{bmatrix}$$

    (b)

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{5}{8} & -\frac{1}{8} & -\frac{1}{8} \\ \frac{1}{8} & -\frac{5}{8} & \frac{3}{8} \end{bmatrix}$$