

11. The result follows from $\det AB = \det A \cdot \det B$ and Theorem 6.17.
12. (a) The solution is $x_1 = 0$, $x_2 = 10$, and $x_3 = 26$.
 (b) We have $D_1 = -1$, $D_2 = 3$, $D_3 = 7$, and $D = 0$, and there are no solutions.
 (c) We have $D_1 = D_2 = D_3 = D = 0$, and there are infinitely many solutions.
 (d) Cramer's rule requires 39 Multiplications/Divisions and 20 Additions/Subtractions.
13. (a) If D_i is the determinant of the matrix formed by replacing the i th column of A with \mathbf{b} and if $D = \det A$, then

$$x_i = D_i/D, \text{ for } i = 1, \dots, n.$$

- (b) $(n+1)! \left(\sum_{k=1}^{n-1} \frac{1}{k!} \right) + n$ multiplications/divisions;
 $(n+1)! - n - 1$ additions/subtractions.

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1. The solutions to the linear systems are as follows.

(a) $x_1 = -3$, $x_2 = 3$, $x_3 = 1$

(b) $x_1 = \frac{1}{2}$, $x_2 = -\frac{9}{2}$, $x_3 = \frac{7}{2}$

2. The solutions to the linear systems are as follows.

(a) $x_1 = 11/20$, $x_2 = 3/10$, $x_3 = 2/5$

(b) $x_1 = 176$, $x_2 = -50$, $x_3 = 24$

3. (a) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

4. (a) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$(c) \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad (d) \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$5. \quad (a) \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 4.5 & 7.5 \\ 0 & 0 & -4 \end{bmatrix}$$

$$(b) \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -2.106719 & 1 & 0 \\ 3.067193 & 1.197756 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1.012 & -2.132 & 3.104 \\ 0 & -0.3955257 & -0.4737443 \\ 0 & 0 & -8.939141 \end{bmatrix}$$

$$(c) \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 1 & -1.33333 & 2 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(d) \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.849190 & 1 & 0 & 0 \\ -0.4596433 & -0.2501219 & 1 & 0 \\ 2.768661 & -0.3079435 & -5.352283 & 1 \end{bmatrix}$$

and

$$U = \begin{bmatrix} 2.175600 & 4.023099 & -2.173199 & 5.196700 \\ 0 & 13.43947 & -4.018660 & 10.80698 \\ 0 & 0 & -0.8929510 & 5.091692 \\ 0 & 0 & 0 & 12.03614 \end{bmatrix}$$

$$6. \quad (a) \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1/2 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$(b) \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 6/5 & -38/11 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1/3 & 1/2 & -1/4 \\ 0 & 11/30 & 21/40 \\ 0 & 0 & 241/88 \end{bmatrix}$$

$$(c) \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 1 & -6/7 & 1 & 0 \\ -1 & 6/7 & -4/25 & 1 \end{bmatrix}$$

and

$$U = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 7/2 & 3 & 0 \\ 0 & 0 & 25/7 & 4 \\ 0 & 0 & 0 & 141/25 \end{bmatrix}$$

$$(d) \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.606196 & -0.168465 & 1 & 0 \\ 0.413289 & 0.778816 & -0.707723 & 1 \end{bmatrix}$$

and

$$U = \begin{bmatrix} 5.1312 & 1.414 & 3.141 & 0 \\ 0 & 5.193 & -2.197 & 5.92056 \\ 0 & 0 & 4.25195 & 4 \\ 0 & 0 & 0 & 12.6828 \end{bmatrix}$$

7. The modified LU algorithm gives the following:

- (a) $x_1 = 1, x_2 = 2, x_3 = -1$ (b) $x_1 = 1, x_2 = 1, x_3 = 1$
 (c) $x_1 = 1.5, x_2 = 2, x_3 = -1.199998, x_4 = 3$
 (d) $x_1 = 2.939851, x_2 = 0.07067770, x_3 = 5.677735, x_4 = 4.379812$

8. The modified LU algorithm gives the following:

- (a) $x_1 = -12, x_2 = -14, x_3 = 17$
 (b) $x_1 = -495/241, x_2 = 840/241, x_3 = 56/241$
 (c) $x_1 = -29/47, x_2 = 58/47, x_3 = 32/141, x_4 = 52/141$
 (d) $x_1 = -0.706123, x_2 = -0.187410, x_3 = 0.569188, x_4 = 0.528704$

$$9. \quad (a) \quad P^t LU = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$$

$$(b) \quad P^t LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 6 \\ 0 & 0 & 4 \end{bmatrix}$$

$$(c) \quad P^t LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -2 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$(d) \quad P^t LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -3 & -1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10. (a) To compute $P^t LU$ requires $\frac{1}{3}n^3 - \frac{1}{3}n$ Multiplications/Divisions and $\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$ Additions/Subtractions.
 (b) If \tilde{P} is obtained from P by a simple row interchange, then $\det \tilde{P} = -\det P$. Thus, if \tilde{P} is obtained from P by k interchanges, we have $\det \tilde{P} = (-1)^k \det P$.

- (c) Only $n - 1$ multiplications are needed in addition to the operations in part (a).
 (d) We have $\det A = -741$. Factoring and computing $\det A$ requires 75 Multiplications/Divisions and 55 Additions/Subtractions.

11. (a) The steps in Algorithm 6.4 give the following:

	Multiplications/Divisions	Additions/Subtractions
Step 2	$n - 1$	0
Step 4	$\sum_{i=2}^{n-1} i - 1$	$\sum_{i=2}^{n-1} i - 1$
Step 5	$\sum_{i=2}^{n-1} \sum_{j=i+1}^n [2(i-1) + 1]$	$\sum_{i=2}^{n-1} \sum_{j=i+1}^n 2(i-1)$
Step 6	$n - 1$	$n - 1$
Totals	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$

- (b) The equations are given by

$$y_1 = \frac{b_1}{l_{11}} \quad \text{and} \quad y_i = b_i - \sum_{j=1}^{i-1} \frac{l_{ij}y_j}{l_{ii}}, \quad \text{for } i = 2, \dots, n.$$

If we assume that $l_{ii} = 1$, for each $i = 1, 2, \dots, n$, then the number of Multiplications/Divisions is

$$\sum_{i=2}^n (i-1) = \frac{n(n-1)}{2},$$

and the number of Additions/Subtractions is the same.

(c)

	Multiplications/divisions	Additions/subtractions
Factoring into LU	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$
Solving $Ly = b$	$\frac{1}{2}n^2 - \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$
Solving $Ux = y$	$\frac{1}{2}n^2 + \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$
Total	$\frac{1}{3}n^3 + n^2 - \frac{1}{3}n$	$\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n$

(d)

	Multiplications/divisions	Additions/subtractions
Factoring into LU	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$
Solving $Ly^{(k)} = b^{(k)}$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$
Solving $Ux^{(k)} = y^{(k)}$	$(\frac{1}{2}n^2 + \frac{1}{2}n)m$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$
Total	$\frac{1}{3}n^3 + mn^2 - \frac{1}{3}n$	$\frac{1}{3}n^3 + (m - \frac{1}{2})n^2 - (m - \frac{1}{6})n$