

the error term is

$$-3h^5 f^{(4)}(\xi)/80.$$

Also,

$$a_i = \int_{x_0}^{x_3} \prod_{\substack{j=0 \\ j \neq i}}^3 \frac{x - x_j}{x_i - x_j} dx, \quad \text{for each } i = 0, 1, 2, 3.$$

Using the change of variables $x = x_0 + th$ gives

$$a_i = h \int_0^3 \prod_{\substack{j=0 \\ j \neq i}}^3 \frac{t - j}{i - j} dt, \quad \text{for each } i = 0, 1, 2, 3.$$

Evaluating the integrals gives $a_0 = \frac{3h}{8}$, $a_1 = \frac{9h}{8}$, $a_2 = \frac{9h}{8}$, and $a_3 = \frac{3h}{8}$.

Exercise Set 4.4, page 203

1. The Composite Trapezoidal rule approximations are:

- (a) 0.639900 (b) 31.3653 (c) 0.784241 (d) -6.42872
 (e) -13.5760 (f) 0.476977 (g) 0.605498 (h) 0.970926

- 2.

	Composite Trapezoidal Approximation	Actual Integral
(a)	0.91193343	0.92073549
(b)	0.09363001	0.08802039
(c)	-0.66468785	-0.66293045
(d)	0.36487225	0.36423547

3. The Composite Simpson's rule approximations are:

- (a) 0.99999998 (b) 1.99999999 (c) 2.2751458 (d) -19.646796

- 4.

	Composite Simpson's Approximation	Actual Integral
(a)	0.92088605	0.92073549
(b)	0.08809221	0.08802039
(c)	-0.66292308	-0.66293045
(d)	0.36423967	0.36423547

5. The Composite Midpoint rule approximations are:

- (a) 0.633096 (b) 11.1568 (c) 0.786700 (d) -6.11274
 (e) -14.9985 (f) 0.478751 (g) 0.602961 (h) 0.947868

6.

	Composite Trapezoidal Approximation	Actual Integral
(a)	0.92862909	0.92073549
(b)	0.08177145	0.08802039
(c)	-0.66067279	-0.66293045
(d)	0.36342511	0.36423547

7. (a) The Composite Trapezoidal rule approximation is 3.15947567.
 (b) The Composite Simpson's rule approximation is 3.10933713.
 (c) The Composite Midpoint rule approximation is 3.00906003.
8. (a) The Composite Trapezoidal rule approximation is 0.4215820.
 (b) The Composite Simpson's rule approximation is 0.4227162.
 (c) The Composite Midpoint rule approximation is 0.4249845.
9. $\alpha = 0.75$
10. $f(-1) = 1$, $f(-0.5) = 2$, $f(0) = 6$, $f(0.5) = 3$, $f(1) = 1$
11. (a) The Composite Trapezoidal rule requires $h < 0.000922295$ and $n \geq 2168$.
 (b) The Composite Simpson's rule requires $h < 0.037658$ and $n \geq 54$.
 (c) The Composite Midpoint rule requires $h < 0.00065216$ and $n \geq 3066$.
12. (a) The Composite Trapezoidal rule requires $h < 0.0069669$ and $n \geq 451$.
 (b) The Composite Simpson's rule requires $h < 0.132749$ and $n \geq 24$.
 (c) The Composite Midpoint rule requires $h < 0.0049263$ and $n \geq 636$.
13. (a) The Composite Trapezoidal rule requires $h < 0.04382$ and $n \geq 46$. The approximation is 0.405471.
 (b) The Composite Simpson's rule requires $h < 0.44267$ and $n \geq 6$. The approximation is 0.405466.
 (c) The Composite Midpoint rule requires $h < 0.03098$ and $n \geq 64$. The approximation is 0.405460.
14. (a) The Composite Trapezoidal rule requires $h < 0.01095$ and $n \geq 91$. With $n = 91$, the approximation is 0.6363013.

- (b) The Composite Simpson's rule requires $h < 0.173205$ and $n \geq 6$. With $n = 6$, the approximation is 0.6362975.
- (c) The Composite Midpoint rule requires $h < 0.0077460$ and $n > 128$. With $n = 130$, the approximation is 0.6362875.
15. (a) Because the right and left limits at 0.1 and 0.2 for f , f' , and f'' are the same, the functions are continuous on $[0, 0.3]$. However,

$$f'''(x) = \begin{cases} 6, & 0 \leq x \leq 0.1 \\ 12, & 0.1 < x \leq 0.2 \\ 12, & 0.2 < x \leq 0.3 \end{cases}$$

is discontinuous at $x = 0.1$.

- (b) We have 0.302506 with an error bound of 1.9×10^{-4} .
- (c) We have 0.302425, and the value of the actual integral is the same.
16. To show that the sum

$$\sum_{j=1}^{n/2} f^{(4)}(\xi_j) 2h$$

is a Riemann Sum, let $y_i = x_{2i}$, for $i = 0, 1, \dots, \frac{n}{2}$. Then $\Delta y_i = y_{i+1} - y_i = 2h$ and $y_{i-1} \leq \xi_i \leq y_i$. Thus,

$$\sum_{j=1}^{n/2} f^{(4)}(\xi_j) \Delta y_j = \sum_{j=1}^{n/2} f^{(4)}(\xi_j) 2h$$

is a Riemann Sum for $\int_a^b f^{(4)}(x) dx$. Hence,

$$\begin{aligned} E(f) &= -\frac{h^5}{90} \sum_{j=1}^{n/2} f^{(4)}(\xi_j) = -\frac{h^4}{180} \left[\sum_{j=1}^{n/2} f^{(4)}(\xi_j) 2h \right] \\ &\approx -\frac{h^4}{180} \int_a^b f^{(4)}(x) dx = -\frac{h^4}{180} [f'''(b) - f'''(a)]. \end{aligned}$$

17. (a) For the Composite Trapezoidal rule, we have

$$E(f) = -\frac{h^3}{12} \sum_{j=1}^n f''(\xi_j) = -\frac{h^2}{12} \sum_{j=1}^n f''(\xi_j) h = -\frac{h^2}{12} \sum_{j=1}^n f''(\xi_j) \Delta x_j,$$

where $\Delta x_j = x_{j+1} - x_j = h$ for each j . Since $\sum_{j=1}^n f''(\xi_j) \Delta x_j$ is a Riemann sum for $\int_a^b f''(x) dx = f'(b) - f'(a)$, we have

$$E(f) \approx -\frac{h^2}{12} [f'(b) - f'(a)].$$

(b) For the Composite Midpoint rule, we have

$$E(f) = \frac{h^3}{3} \sum_{j=1}^{n/2} f''(\xi_j) = \frac{h^2}{6} \sum_{j=1}^{n/2} f''(\xi_j)(2h).$$

But $\sum_{j=1}^{n/2} f''(\xi_j)(2h)$ is a Riemann sum for $\int_a^b f''(x) dx = f'(b) - f'(a)$, so

$$E(f) \approx \frac{h^2}{6} [f'(b) - f'(a)].$$

18. (a) Composite Trapezoidal Rule: With $h = 0.0069669$, the error estimate is 2.541×10^{-5} .
 (b) Composite Simpson's Rule: With $h = 0.132749$, the error estimate is 3.252×10^{-5} .
 (c) Composite Midpoint Rule: With $h = 0.0049263$, the error estimate is 2.541×10^{-5} .
19. (a) The estimate using the Composite Trapezoidal rule is $-\frac{1}{2}h^2 \ln 2 = -6.296 \times 10^{-6}$.
 (b) The estimate using the Composite Simpson's rule is $-\frac{1}{240}h^2 = -3.75 \times 10^{-6}$.
 (c) The estimate using the Composite Midpoint rule is $\frac{1}{6}h^2 \ln 2 = 6.932 \times 10^{-6}$.
20. (a) 0.68269822 obtained using $n = 10$ in Composite Simpson's rule.
 (b) 0.95449101 obtained using $n = 14$ in Composite Simpson's rule.
 (c) 0.99729312 obtained using $n = 20$ in Composite Simpson's rule.
21. The length is approximately 15.8655.
22. The length of the track is approximately 9858 ft.
23. Composite Simpson's rule with $h = 0.25$ gives 2.61972 s.
24. An approximation for T is 1054.694.
25. The length is approximately 58.47082, using $n = 100$ in the Composite Simpson's rule.
26. (a) For $p_0 = 0.5$, we have $p_6 = 1.644854$ with $n = 20$.
 (b) For $p_0 = 0.5$, we have $p_6 = 1.645085$ with $n = 40$.

Exercise Set 4.5, page 211

1. Romberg integration gives $R_{3,3}$ as follows:

- | | | | |
|---------------|----------------|----------------|----------------|
| (a) 0.1922593 | (b) 0.1606105 | (c) -0.1768200 | (d) 0.08875677 |
| (e) 2.5879685 | (f) -0.7341567 | (g) 0.6362135 | (h) 0.6426970 |