4.

$\times a)$ $f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f^{(3)}(x) + \frac{h^4}{12}f^{(4)}(x) + \cdots\cdots$

$\times b)$ $f(x) = f(x)$

$\times c)$ $f(x+2h) = f(x) + 2hf'(x) + 2h^2 f''(x) + \frac{4}{3}h^3 f^{(3)}(x) + \frac{2}{3}f^{(4)}(x) + \cdots\cdots$

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(1) $\underline{f'(x)}$:

$$\begin{cases} a+b+c=0 \\ -a \quad +2c = 1 \\ \frac{a}{2} \quad +2c = 0 \end{cases} \Rightarrow \begin{cases} a = -\frac{2}{3} \\ b = \frac{1}{2} \\ c = \frac{1}{6} \end{cases}$$

$$\Rightarrow \frac{f(x+2h)+3f(x)-4f(x-h)}{6h} = f'(x) + h^2\left(\frac{2}{9}f^{(3)}(\xi_2) + \frac{1}{9}f^{(3)}(\xi_1)\right)$$

, where $x-h<\xi_1<x$, $x<\xi_2<x+2h$

and $\because$ $\frac{3}{9}\min\{f^{(3)}(\xi_2), f^{(3)}(\xi_1)\} \le \frac{2}{9}f^{(3)}(\xi_2) + \frac{1}{9}f^{(3)}(\xi_1) \le \frac{3}{9}\max\{f^{(3)}(\xi_2), f^{(3)}(\xi_1)\}$

By intermediate value theorem, $\exists$ ξ between ξ_1 and ξ_2

such that $\frac{3}{9}f^{(3)}(\xi) = \frac{2}{9}f^{(3)}(\xi_2) + \frac{1}{9}f^{(3)}(\xi_1)$

$$\Rightarrow \left|\frac{f(x+2h)+3f(x)-4f(x-h)}{6h} - f'(x)\right| = \frac{3}{9}|f^{(3)}(\xi)|h^2 \quad ※$$

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(2) $f''(x)$:

$$\begin{cases} a+b+c=0 \\ -a \quad +2c = 0 \\ \frac{a}{2} \quad +2c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{2}{3} \\ b = -1 \\ c = \frac{1}{3} \end{cases}$$

$$\Rightarrow \frac{f(x+2h)-3f(x)+2f(x-h)}{3h^2} = f''(x) + h^2\left(\frac{4}{9}f^{(3)}(\xi_2') - \frac{1}{9}f^{(3)}(\xi_1')\right)$$

$$\Rightarrow \left|\frac{f(x+2h)-3f(x)+2f(x-h)}{3h^2} - f''(x)\right| \le \left|\frac{4}{9}f^{(3)}(\xi_2') - \frac{1}{9}f^{(3)}(\xi_1')\right|h^2$$

$$\le \frac{5}{9}\max_{\xi}|f^{(3)}(\xi)|h^2 \quad ※$$