2. $$\left|\prod_{n=1}^{N}\frac{4n^2}{4n^2-1}-\prod_{n=1}^{N+M}\frac{4n^2}{4n^2-1}\right| = \left|\prod_{n=1}^{N}-\prod_{n=1}^{N+1}+\prod_{n=1}^{N+1}-\prod_{n=1}^{N+2}+\prod_{n=1}^{N+2}+\cdots\cdots-\prod_{n=1}^{N+M}\right|$$

$$\leq \sum_{K=N}^{N+M}\left|\prod_{n=1}^{K}\frac{4n^2}{4n^2-1}-\prod_{n=1}^{K+1}\frac{4n^2}{4n^2-1}\right|$$

$$\because \lim_{M\to\infty}\left|\prod_{n=1}^{N}\frac{4n^2}{4n^2-1}-\prod_{n=1}^{N+M}\frac{4n^2}{4n^2-1}\right| = \left|\prod_{n=1}^{N}\frac{4n^2}{4n^2-1}-\frac{\pi}{2}\right|$$

$$\Rightarrow \left|\frac{\Pi_N}{2}-\frac{\pi}{2}\right| \leq \sum_{K=N}^{\infty}\left|\prod_{n=1}^{K}\frac{4n^2}{4n^2-1}-\prod_{n=1}^{K+1}\frac{4n^2}{4n^2-1}\right|$$

$$= \sum_{K=N}^{\infty}\left(\prod_{n=1}^{K}\frac{4n^2}{4n^2-1}\right)\cdot\left(\left|1-\frac{4(K+1)^2}{4(K+1)^2-1}\right|\right)$$

$$= \sum_{K=N}^{\infty}\left(\prod_{n=1}^{K}\frac{4n^2}{4n^2-1}\right)\cdot\left(\left|\frac{-1}{4(K+1)^2-1}\right|\right)$$

$$\leq \sum_{K=N}^{\infty}\frac{\pi}{2}\cdot\frac{1}{4K^2}$$

$$= \frac{\pi}{8}\underbrace{\sum_{K=N}^{\infty}\frac{1}{K^2}}_{\text{下和}} \leq \frac{\pi}{8}\int_{N-1}^{\infty}\frac{1}{x^2}\,dx = \frac{\pi}{8}\cdot\frac{1}{N-1}$$

$\therefore$ $\frac{\Pi_N}{2}$ converges to $\frac{\pi}{2}$ with rate of convergence $O(\frac{1}{N})$