

Exercise Set 1.2, page 26

1.

	Absolute error	Relative error
(a)	0.001264	4.025×10^{-4}
(b)	7.346×10^{-6}	2.338×10^{-6}
(c)	2.818×10^{-4}	1.037×10^{-4}
(d)	2.136×10^{-4}	1.510×10^{-4}
(e)	2.647×10^1	1.202×10^{-3}
(f)	1.454×10^1	1.050×10^{-2}
(g)	420	1.042×10^{-2}
(h)	3.343×10^3	9.213×10^{-3}

2. The largest intervals are:

- (a) (3.1412784, 3.1419068) (b) (2.7180100, 2.7185536)
- (c) (1.4140721, 1.4143549) (d) (1.9127398, 1.9131224)

3. The largest intervals are

- (a) (149.85, 150.15) (b) (899.1, 900.9)
- (c) (1498.5, 1501.5) (d) (89.91, 90.09)

4. The calculations and their errors are:

- (a) (i) 17/15 (ii) 1.13 (iii) 1.13 (iv) both 3×10^{-3}
- (b) (i) 4/15 (ii) 0.266 (iii) 0.266 (iv) both 2.5×10^{-3}
- (c) (i) 139/660 (ii) 0.211 (iii) 0.210 (iv) 2×10^{-3} , 3×10^{-3}
- (d) (i) 301/660 (ii) 0.455 (iii) 0.456 (iv) 2×10^{-3} , 1×10^{-4}

5.

	Approximation	Absolute error	Relative error
(a)	134	0.079	5.90×10^{-4}
(b)	133	0.499	3.77×10^{-3}
(c)	2.00	0.327	0.195
(d)	1.67	0.003	1.79×10^{-3}
(e)	1.80	0.154	0.0786
(f)	-15.1	0.0546	3.60×10^{-3}
(g)	0.286	2.86×10^{-4}	10^{-3}
(h)	0.00	0.0215	1.00

6.

	Approximation	Absolute error	Relative error
(a)	133.9	0.021	1.568×10^{-4}
(b)	132.5	0.001	7.55×10^{-6}
(c)	1.700	0.027	0.01614
(d)	1.673	0	0
(e)	1.986	0.03246	0.01662
(f)	-15.16	0.005377	3.548×10^{-4}
(g)	0.2857	1.429×10^{-5}	5×10^{-5}
(h)	-0.01700	0.0045	0.2092

7.

	Approximation	Absolute error	Relative error
(a)	133	0.921	6.88×10^{-3}
(b)	132	0.501	3.78×10^{-3}
(c)	1.00	0.673	0.402
(d)	1.67	0.003	1.79×10^{-3}
(e)	3.55	1.60	0.817
(f)	-15.2	0.0454	0.00299
(g)	0.284	0.00171	0.00600
(h)	0	0.02150	1

8.

	Approximation	Absolute error	Relative error
(a)	133.9	0.021	1.568×10^{-4}
(b)	132.5	0.001	7.55×10^{-6}
(c)	1.600	0.073	0.04363
(d)	1.673	0	0
(e)	1.983	0.02945	0.01508
(f)	-15.15	0.004622	3.050×10^{-4}
(g)	0.2855	2.143×10^{-4}	7.5×10^{-4}
(h)	-0.01700	0.0045	0.2092

9.

	Approximation	Absolute error	Relative error
(a)	3.14557613	3.983×10^{-3}	1.268×10^{-3}
(b)	3.14162103	2.838×10^{-5}	9.032×10^{-6}

10.

	Approximation	Absolute error	Relative error
(a)	2.7166667	0.0016152	5.9418×10^{-4}
(b)	2.718281801	2.73×10^{-8}	1.00×10^{-8}

11. (a) We have

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x - \sin x} &= \lim_{x \rightarrow 0} \frac{-x \sin x}{1 - \cos x} \\ &= \lim_{x \rightarrow 0} \frac{-\sin x - x \cos x}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{-2 \cos x + x \sin x}{\cos x} = -2 \end{aligned}$$

(b) $f(0.1) \approx -1.941$

(c) $\frac{x(1 - \frac{1}{2}x^2) - (x - \frac{1}{6}x^3)}{x - (x - \frac{1}{6}x^3)} = -2$

(d) The relative error in part (b) is 0.029. The relative error in part (c) is 0.00050.

12. (a) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x} = \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{1} = 2$

(b) $f(0.1) \approx 2.05$

(c) $\frac{1}{x} \left((1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3) - (1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3) \right) = \frac{1}{x} (2x + \frac{1}{3}x^3) = 2 + \frac{1}{3}x^2$; using three-digit rounding arithmetic and $x = 0.1$, we obtain 2.00

(d) The relative error in part (b) is 0.0233. The relative error in part (c) is 0.00166.

13.

	x_1	Absolute error	Relative error	x_2	Absolute error	Relative error
(a)	92.26	0.01542	1.672×10^{-4}	0.005419	6.273×10^{-7}	1.157×10^{-4}
(b)	0.005421	1.264×10^{-6}	2.333×10^{-4}	-92.26	4.580×10^{-3}	4.965×10^{-5}
(c)	10.98	6.875×10^{-3}	6.257×10^{-4}	0.001149	7.566×10^{-8}	6.584×10^{-5}
(d)	-0.001149	7.566×10^{-8}	6.584×10^{-5}	-10.98	6.875×10^{-3}	6.257×10^{-4}

14.

	Approximation for x_1	Absolute error	Relative error
(a)	92.24	0.004580	4.965×10^{-5}
(b)	0.005417	2.736×10^{-6}	5.048×10^{-4}
(c)	10.98	6.875×10^{-3}	6.257×10^{-4}
(d)	-0.001149	7.566×10^{-8}	6.584×10^{-5}

	Approximation for x_2	Absolute error	Relative error
(a)	0.005418	2.373×10^{-6}	4.377×10^{-4}
(b)	-92.25	5.420×10^{-3}	5.875×10^{-5}
(c)	0.001149	7.566×10^{-8}	6.584×10^{-5}
(d)	-10.98	6.875×10^{-3}	6.257×10^{-4}

15. The machine numbers are equivalent to

- (a) 3224 (b) -3224 (c) 1.32421875
 (d) 1.32421875000000002220446049250313080847263336181640625

16. (a) Next Largest: 3224.00000000000045474735088646411895751953125;
 Next Smallest: 3223.99999999999954525264911353588104248046875
 (b) Next Largest: -3224.00000000000045474735088646411895751953125;
 Next Smallest: -3223.99999999999954525264911353588104248046875
 (c) Next Largest: 1.32421875000000002220446049250313080847263336181640625;
 Next Smallest: 1.3242187499999997779553950749686919152736663818359375
 (d) Next Largest: 1.3242187500000000444089209850062616169452667236328125;
 Next Smallest: 1.32421875
17. (b) The first formula gives -0.00658, and the second formula gives -0.0100. The true three-digit value is -0.0116.

18. (a) -1.82 (b) 7.09×10^{-3}

(a) The formula in (b) is more accurate since subtraction is not involved.

19. The approximate solutions to the systems are

- (a) $x = 2.451, y = -1.635$ (b) $x = 507.7, y = 82.00$

20. (a) $x = 2.460 \quad y = -1.634$ (b) $x = 477.0 \quad y = 76.93$

21. (a) In nested form, we have

$$f(x) = (((1.01e^x - 4.62)e^x - 3.11)e^x + 12.2)e^x - 1.99.$$

- (b) -6.79 (c) -7.07

(d) The absolute errors are

$$|-7.61 - (-6.71)| = 0.82 \quad \text{and} \quad |-7.61 - (-7.07)| = 0.54.$$

Nesting is significantly better since the relative errors are

$$\left| \frac{0.82}{-7.61} \right| = 0.108 \quad \text{and} \quad \left| \frac{0.54}{-7.61} \right| = 0.071,$$

22. We have $39.375 \leq \text{Volume} \leq 86.625$ and $71.5 \leq \text{Surface Area} \leq 119.5$.

23. (a) $n = 77$

(b) $n = 35$

24. When $d_{k+1} < 5$,

$$\left| \frac{y - fl(y)}{y} \right| = \frac{0.d_{k+1} \dots \times 10^{n-k}}{0.d_1 \dots \times 10^n} \leq \frac{0.5 \times 10^{-k}}{0.1} = 0.5 \times 10^{-k+1}.$$

When $d_{k+1} > 5$,

$$\left| \frac{y - fl(y)}{y} \right| = \frac{(1 - 0.d_{k+1} \dots) \times 10^{n-k}}{0.d_1 \dots \times 10^n} < \frac{(1 - 0.5) \times 10^{-k}}{0.1} = 0.5 \times 10^{-k+1}.$$

25. (a) $m = 17$

(b)

$$\begin{aligned} \binom{m}{k} &= \frac{m!}{k!(m-k)!} = \frac{m(m-1) \cdots (m-k-1)(m-k)!}{k!(m-k)!} \\ &= \binom{m}{k} \binom{m-1}{k-1} \cdots \binom{m-k-1}{1} \end{aligned}$$

(c) $m = 181707$

(d) 2,597,000; actual error 1960; relative error 7.541×10^{-4}

26. (a) The actual error is $|f'(\xi)\epsilon|$, and the relative error is $|f'(\xi)\epsilon| \cdot |f(x_0)|^{-1}$, where the number ξ is between x_0 and $x_0 + \epsilon$.

(b) (i) 1.4×10^{-5} ; 5.1×10^{-6} (ii) 2.7×10^{-6} ; 3.2×10^{-6}

(c) (i) 1.2; 5.1×10^{-5} (ii) 4.2×10^{-5} ; 7.8×10^{-5}

27. (a) 124.03

(b) 124.03

(c) -124.03

(d) -124.03

(e) 0.0065

(f) 0.0065

(g) -0.0065

(h) -0.0065

28. Since $0.995 \leq P \leq 1.005$, $0.0995 \leq V \leq 0.1005$, $0.082055 \leq R \leq 0.082065$, and $0.004195 \leq N \leq 0.004205$, we have $287.61^\circ \leq T \leq 293.42^\circ$. Note that $15^\circ\text{C} = 288.16\text{K}$.

When P is doubled and V is halved, $1.99 \leq P \leq 2.01$ and $0.0497 \leq V \leq 0.0503$ so that $286.61^\circ \leq T \leq 293.72^\circ$. Note that $19^\circ\text{C} = 292.16\text{K}$. The laboratory figures are within an acceptable range.