Homework Assignment for Week 14

Assigned Dec 16, 2011.

1. Section 7.4: Construct the Hilbert matrix $H^{(n)}$ in problem 11 and compute the condition number $\kappa(H^{(n)})$ for $n = 5, 10, 20, 30, \cdots$, respectively. You can use the matlab/octave built-in function 'cond'. This is a well-known example of ill-conditioned matrix that corresponds to the continuous least square approximation.

2. Do the same for discrete least square approximation. First derive the equation by trying to find $p(x) \in P_n$ that minimizes $\sum_{j=0}^{m}(p(x_j) - f(x_j))^2$ with $m > n$, $x_j = \frac{j}{m}$ and $f(x_j)$ given.

   If time permits, try to find the condition numbers of the corresponding $A^T A$ with $m = n + 1$ and/or $m = 2n$ (in the continuous case, $A^T A$ corresponds to $H^{(n+1)}$ with $m = \infty$ in some sense).

3. Section 2.2: Problems 2(b), 7, 12, 24.

4. The iteration $x_{n+1} = g(x_n)$ almost for sure does not converge when $|g'| > 1$ near the solution x_*. An easy remedy is to give a rough estimate on $g'(x_*)$, then perform the iteration
$$x_{n+1} = \alpha x_n + (1 - \alpha) g(x_n).$$
   Show that this iteration, if convergent, gives a root to $x = g(x)$. The advantage of the new iteration is that, you can choose an α according to the approximate value of $g'(x_*)$, so that the new iteration converges. Apply this trick to the equation $x = g(x) = 1 - 2x + 0.2 \sin x$.