Homework Assignment for Week 10

Assigned Nov 18, 2011.

1. Section 6.6: Problems 3(a), 7(a). (These are paper and pencil assignments).

   For problems 2 and 3, it may be helpful to review the LU decomposition summarized in page 43-45 of Chap 6's slides (2011 version).

2. Find the numbers of multiplications/divisions (to leading order) needed for LDL^T and Choleski decompositions (both without pivoting), respectively.

3. Derive the recursive version of LDL^T decomposition (without pivoting).

4. Discretize the following system

$$\begin{array}{l} u''(x) - \alpha u(x) = \sin(\pi x), \quad \alpha > 0, \quad x \in (0,1) \\ u(0) = 0, \; u(1) = 0 \end{array} \tag{1}$$

   with uniformly spaced grids $0 = x_0 < x_1 < \cdots < x_N = 1$, $x_i - x_{i-1} = h = 1/N$, using second order finite difference method. Let A be the corresponding matrix.

   (a) Show that Gauss elimination for A is identical to Gauss elimination with (scaled) partial pivoting for A (i.e. the diagonal elements are exactly the pivot elements). (Hint: First apply Theorem 6.19.)

   (b) Form the matrix A in matlab/octave. The built-in functions 'diag', 'ones' should be useful.

   (c) Implement Algorithm 6.7 (LU decomposition for tridiagonal system, together with forward and backward substitutions) to solve for u_j. For future purposes, you should separate the LU-decomposition part and forward/backward substitution part as two separate subroutines. In many applications, you only need to LU-decompose once and solve for $u(x_j)$ for many times. The exact solution to (1) is $u_e(x) = -\sin(\pi x)/(\pi^2 + \alpha)$. If you got all things done correctly, you should be able to see numerically that $\sup_j |u_h(x_j) - u_e(x_j)| = O(h^2)$.

   You can also check your answer by the built-in command '$u = A\backslash f$'. However, keep in mind that it is much slower for this problem.

   Keep your code for next week's homework assignment, which will need to be handed in.