

## Quiz 05

Dec 27, 2011.

1. Derive the normal equation for the following overdetermined system and find the solution. Here  $n$  is small so you don't need to worry about the condition number. If you can't derive but memorized it, then solve it directly with partial credit.

$$\begin{aligned}a_1 + 2a_2 &= 0 \\2a_1 - a_2 &= 1 \\3a_1 - a_2 &= 5\end{aligned}$$

2. The sequence  $a_n = 1 + \frac{1}{n^2}$  converges to its limit of order  $\alpha$ . Find  $\alpha$ . Give an example of a quadratically convergent sequence.
3. Give a convergent fixed point iteration (and not other methods) for solving  $f(x) = x - 2 \cos x = 0$ . Explain why your formulae converges. You can use the fact that the solution  $x^* \approx 1.03$ ,  $f'(x^*) \approx 1 + 2 \cdot \sin(1.03) \approx 2.71$ . Write the iteration formula and need not implement.
4. Write a pseudo-code for solving  $f(x) = x - 2 \cos x = 0$  with Newton's method till  $|f(x_n)| < 10^{-10}$ . Do the same for secant method. Need not implement them.
5. Implement the secant method from last problem and report your  $x_k$ ,  $k = 0, \dots, n$ .

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