11. The result follows from $\det AB = \det A \cdot \det B$ and Theorem 6.17.

12. (a) The solution is $x_1 = 0$, $x_2 = 10$, and $x_3 = 26$.
    (b) We have $D_1 = -1$, $D_2 = 3$, $D_3 = 7$, and $D = 0$, and there are no solutions.
    (c) We have $D_1 = D_2 = D_3 = D = 0$, and there are infinitely many solutions.
    (d) Cramer's rule requires 39 Multiplications/Divisions and 20 Additions/Subtractions.

13. (a) If D_i is the determinant of the matrix formed by replacing the ith column of A with $\mathbf{b}$ and if $D = \det A$, then

$$x_i = D_i/D, \text{ for } i = 1, \ldots, n.$$

    (b) $(n+1)!\left(\sum_{k=1}^{n-1} \frac{1}{k!}\right) + n$ multiplications/divisions;
    $(n+1)! - n - 1$ additions/subtractions.

Exercise Set 6.5, page 395

1. The solutions to the linear systems are as follows.

   (a) $x_1 = -3$, $x_2 = 3$, $x_3 = 1$ (b) $x_1 = \frac{1}{2}$, $x_2 = -\frac{9}{2}$, $x_3 = \frac{7}{2}$

2. The solutions to the linear systems are as follows.

   (a) $x_1 = 11/20$, $x_2 = 3/10$, $x_3 = 2/5$ (b) $x_1 = 176$, $x_2 = -50$, $x_3 = 24$

3. (a) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (c) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

4. (a) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ (d) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. (a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 1.5 & 1 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 4.5 & 7.5 \\ 0 & 0 & -4 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ -2.106719 & 1 & 0 \\ 3.067193 & 1.197756 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1.012 & -2.132 & 3.104 \\ 0 & -0.3955257 & -0.4737443 \\ 0 & 0 & -8.939141 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 1 & -1.33333 & 2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.849190 & 1 & 0 & 0 \\ -0.4596433 & -0.2501219 & 1 & 0 \\ 2.768661 & -0.3079435 & -5.352283 & 1 \end{bmatrix}$

and

$$U = \begin{bmatrix} 2.175600 & 4.023099 & -2.173199 & 5.196700 \\ 0 & 13.43947 & -4.018660 & 10.80698 \\ 0 & 0 & -0.8929510 & 5.091692 \\ 0 & 0 & 0 & 12.03614 \end{bmatrix}$$

6. (a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1/2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 3 \\ 0 & 0 & 1/2 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 & 0 \\ 3/5 & 1 & 0 \\ 6/5 & -38/11 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1/3 & 1/2 & -1/4 \\ 0 & 11/30 & 21/40 \\ 0 & 0 & 241/88 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 1 & -6/7 & 1 & 0 \\ -1 & 6/7 & -4/25 & 1 \end{bmatrix}$

and

$$U = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 7/2 & 3 & 0 \\ 0 & 0 & 25/7 & 4 \\ 0 & 0 & 0 & 141/25 \end{bmatrix}$$

(d) $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.606196 & -0.168465 & 1 & 0 \\ 0.413289 & 0.778816 & -0.707723 & 1 \end{bmatrix}$

and

$$U = \begin{bmatrix} 5.1312 & 1.414 & 3.141 & 0 \\ 0 & 5.193 & -2.197 & 5.92056 \\ 0 & 0 & 4.25195 & 4 \\ 0 & 0 & 0 & 12.6828 \end{bmatrix}$$

7. The modified LU algorithm gives the following:

(a) $x_1 = 1$, $x_2 = 2$, $x_3 = -1$

(b) $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

(c) $x_1 = 1.5$, $x_2 = 2$, $x_3 = -1.199998$, $x_4 = 3$

(d) $x_1 = 2.939851$, $x_2 = 0.07067770$, $x_3 = 5.677735$, $x_4 = 4.379812$

8. The modified LU algorithm gives the following:

(a) $x_1 = -12$, $x_2 = -14$, $x_3 = 17$

(b) $x_1 = -495/241$, $x_2 = 840/241$, $x_3 = 56/241$

(c) $x_1 = -29/47$, $x_2 = 58/47$, $x_3 = 32/141$, $x_4 = 52/141$

(d) $x_1 = -0.706123$, $x_2 = -0.187410$, $x_3 = 0.569188$, $x_4 = 0.528704$

9. (a) $P^tLU = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$

(b) $P^tLU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 6 \\ 0 & 0 & 4 \end{bmatrix}$

(c) $P^tLU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -2 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

(d) $P^tLU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 5 & -3 & -1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

10. (a) To compute P^tLU requires $\frac{1}{3}n^3 - \frac{1}{3}n$ Multiplications/Divisions and $\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$ Additions/Subtractions.

(b) If $\tilde{P}$ is obtained from P by a simple row interchange, then $\det \tilde{P} = -\det P$. Thus, if $\tilde{P}$ is obtained from P by k interchanges, we have $\det \tilde{P} = (-1)^k \det P$.

(c) Only $n-1$ multiplications are needed in addition to the operations in part (a).

(d) We have $\det A = -741$. Factoring and computing $\det A$ requires 75 Multiplications/Divisions and 55 Additions/Subtractions.

11. (a) The steps in Algorithm 6.4 give the following:

	Multiplications/Divisions	Additions/Subtractions
Step 2	$n-1$	0
Step 4	$\sum_{i=2}^{n-1} i-1$	$\sum_{i=2}^{n-1} i-1$
Step 5	$\sum_{i=2}^{n-1}\sum_{j=i+1}^{n}[2(i-1)+1]$	$\sum_{i=2}^{n-1}\sum_{j=i+1}^{n}2(i-1)$
Step 6	$n-1$	$n-1$
Totals	$\frac{1}{3}n^3-\frac{1}{3}n$	$\frac{1}{3}n^3-\frac{1}{2}n^2+\frac{1}{6}n$

(b) The equations are given by

$$y_1 = \frac{b_1}{l_{11}} \quad \text{and} \quad y_i = b_i - \sum_{j=1}^{i-1}\frac{l_{ij}y_j}{l_{ii}}, \quad \text{for } i = 2,\ldots,n.$$

If we assume that $l_{ii} = 1$, for each $i = 1,2,\ldots,n$, then the number of Multiplications/Divisions is

$$\sum_{i=2}^{n}(i-1) = \frac{n(n-1)}{2},$$

and the number of Additions/Subtractions is the same.

(c)

	Multiplications/divisions	Additions/subtractions
Factoring into LU	$\frac{1}{3}n^3-\frac{1}{3}n$	$\frac{1}{3}n^3-\frac{1}{2}n^2+\frac{1}{6}n$
Solving $Ly=b$	$\frac{1}{2}n^2-\frac{1}{2}n$	$\frac{1}{2}n^2-\frac{1}{2}n$
Solving $Ux=y$	$\frac{1}{2}n^2+\frac{1}{2}n$	$\frac{1}{2}n^2-\frac{1}{2}n$
Total	$\frac{1}{3}n^3+n^2-\frac{1}{3}n$	$\frac{1}{3}n^3+\frac{1}{2}n^2-\frac{5}{6}n$

(d)

	Multiplications/divisions	Additions/subtractions
Factoring into LU	$\frac{1}{3}n^3-\frac{1}{3}n$	$\frac{1}{3}n^3-\frac{1}{2}n^2+\frac{1}{6}n$
Solving $Ly^{(k)}=b^{(k)}$	$(\frac{1}{2}n^2-\frac{1}{2}n)m$	$(\frac{1}{2}n^2-\frac{1}{2}n)m$
Solving $Ux^{(k)}=y^{(k)}$	$(\frac{1}{2}n^2+\frac{1}{2}n)m$	$(\frac{1}{2}n^2-\frac{1}{2}n)m$
Total	$\frac{1}{3}n^3+mn^2-\frac{1}{3}n$	$\frac{1}{3}n^3+(m-\frac{1}{2})n^2-(m-\frac{1}{6})n$