and

$$\left|\int_a^b f(x)\,dx - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right)\right| \approx \frac{h^3}{48}\left|f''(\mu)\right|.$$

So

$$\left|\int_a^b f(x)\,dx - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right)\right| \approx \frac{1}{3}\left|T(a,b) - T\left(a, \frac{a+b}{2}\right) - T\left(\frac{a+b}{2}, b\right)\right|.$$

10. For t between o and 1 we have the following values.

t	$c(t)$	$s(t)$
0.1	0.0999975	0.000523589
0.2	0.199921	0.00418759
0.3	0.299399	0.0141166
0.4	0.397475	0.0333568
0.5	0.492327	0.0647203
0.6	0.581061	0.110498
0.7	0.659650	0.172129
0.8	0.722844	0.249325
0.9	0.764972	0.339747
1.0	0.779880	0.438245

Exercise Set 4.7, page 226

1. Gaussian quadrature gives:

(a) 0.1922687 (b) 0.1594104 (c) −0.1768190 (d) 0.08926302

(e) 2.5913247 (f) −0.7307230 (g) 0.6361966 (h) 0.6423172

2. Gaussian quadrature with $n = 3$ gives:

(a) 0.1922594 (b) 0.1605954 (c) −0.1768200 (d) 0.08875385

(e) 2.5892580 (f) −0.7337990 (g) 0.6362132 (h) 0.6427011

3. Gaussian quadrature gives:

(a) 0.1922594 (b) 0.1606028 (c) −0.1768200 (d) 0.08875529

(e) 2.5886327 (f) −0.7339604 (g) 0.6362133 (h) 0.6426991

4. Gaussian quadrature with $n = 5$ gives:

(a) 0.1922594 (b) 0.1606028 (c) −0.1768200 (d) 0.08875528

(e) 2.5886286 (f) −0.7339687 (g) 0.6362133 (h) 0.6426991

5. $a = 1,\ b = 1,\ c = \frac{1}{3},\ d = -\frac{1}{3}$

6. $a = \frac{7}{15},\ b = \frac{16}{15},\ c = \frac{7}{15},\ d = \frac{1}{15},\ e = -\frac{1}{15}$

7. The Legendre polynomials $P_2(x)$ and $P_3(x)$ are given by

$$P_2(x) = \frac{1}{2}\left(3x^2 - 1\right) \quad \text{and} \quad P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right),$$

so their roots are easily verified.
For $n = 2$,

$$c_1 = \int_{-1}^{1} \frac{x + 0.5773502692}{1.1547005}\, dx = 1$$

and

$$c_2 = \int_{-1}^{1} \frac{x - 0.5773502692}{-1.1547005}\, dx = 1.$$

For $n = 3$,

$$c_1 = \int_{-1}^{1} \frac{x(x + 0.7745966692)}{1.2}\, dx = \frac{5}{9},$$

$$c_2 = \int_{-1}^{1} \frac{(x + 0.7745966692)(x - 0.7745966692)}{-0.6}\, dx = \frac{8}{9},$$

and

$$c_3 = \int_{-1}^{1} \frac{x(x - 0.7745966692)}{1.2}\, dx = \frac{5}{9}.$$

8. Let $P(x) = \prod_{i=1}^{n}(x - x_i)^2$. Then $Q(P) = 0$ and $\int_{-1}^{1} P(x)\, dx \neq 0$.