33. (a) (i) $B_3(x) = x$ (ii) $B_3(x) = 1$ (d) $n \geq 250{,}000$

Exercise Set 3.2, page 127

1. The interpolating polynomials are as follows.

   (a) $P_1(x) = 16.9441 + 3.1041(x - 8.1); P_1(8.4) = 17.87533$
   $P_2(x) = P_1(x) + 0.06(x - 8.1)(x - 8.3); P_2(8.4) = 17.87713$
   $P_3(x) = P_2(x) + -0.00208333(x - 8.1)(x - 8.3)(x - 8.6); P_3(8.4) = 17.87714$

   (b) $P_1(x) = -0.1769446 + 1.9069687(x - 0.6); P_1(0.9) = 0.395146$
   $P_2(x) = P_1(x) + 0.959224(x - 0.6)(x - 0.7); P_2(0.9) = 0.4526995$
   $P_3(x) = P_2(x) - 1.785741(x - 0.6)(x - 0.7)(x - 0.8); P_3(0.9) = 0.4419850$

2. The interpolating polynomials are as follows.

   (a) $P_1(x) = 1.0 + 2.594880000x; P_1(0.43) = 2.115798400$
   $P_2(x) = P_1(x) + 3.366720000x(x - 0.25); P_2(0.43) = 2.376382528$
   $P_3(x) = P_2(x) + 2.912106667x(x - 0.25)(x - 0.5); P_3(0.43) = 2.360604734$

   (b) $P_1(x) = 0.726560000 - 2.421880000x; P_1(0) = 0.726560000$
   $P_2(x) = P_1(x) + 1.812509333(x + 0.5)(x + 0.25); P_2(0) = 0.9531236666$
   $P_3(x) = P_2(x) - 1.000010666(x + 0.5)(x + 0.25)(x - 0.25); P_3(0) = 0.9843739999$

3. In the following equations, we have $s = (1/h)\,(x - x_0)$.

   (a) $P_1(s) = -0.718125 - 0.0470625s;\ P_1\left(-\frac{1}{3}\right) = -0.006625$
   $P_2(s) = P_1(s) + 0.312625s(s - 1)/2;\ P_2\left(-\frac{1}{3}\right) = 0.1803056$
   $P_3(s) = P_2(s) + 0.09375s(s - 1)(s - 2)/6;\ P_3\left(-\frac{1}{3}\right) = 0.1745185$

   (b) $P_1(s) = -0.62049958 + 0.3365129s;\ P_1(0.25) = -0.1157302$
   $P_2(s) = P_1(s) - 0.04592527s(s - 1)/2;\ P_2(0.25) = -0.1329522$
   $P_3(s) = P_2(s) - 0.00283891s(s - 1)(s - 2)/6;\ P_3(0.25) = -0.1327748$

4. In the following equations, we have $s = (1/h)\,(x - x_0)$.

   (a) $P_1(s) = 1.0 + 0.6487200000s;\ P_1(0.43) = 2.115798400$
   $P_2(s) = P_1(s) + 0.2104200000s(s - 1);\ P_2(0.43) = 2.376382528$
   $P_3(s) = P_2(s) + 0.04550166667s(s - 1)(s - 2);\ P_3(0.43) = 2.360604734$

   (b) $P_1(s) = -0.29004986 - 0.2707474800s;\ P_1(0.18) = -0.5066478440$
   $P_2(s) = P_1(s) + 0.008762550000s(s - 1);\ P_2(0.18) = -0.5080498520$
   $P_3(s) = P_2(s) - 0.0004855333333s(s - 1)(s - 2);\ P_3(0.18) = -0.5081430744$

5. In the following equations, we have $s = (1/h)\,(x - x_n)$.

   (a) $P_1(s) = 1.101+0.7660625s;\quad f(-\frac{1}{3}) \approx P_1(-\frac{4}{3}) = 0.07958333$ $P_2(s) = P_1(s)+0.406375s(s+1)/2;\quad f(-\frac{1}{3}) \approx P_2(-\frac{4}{3}) = 0.1698889$ $P_3(s) = P_2(s)+0.09375s(s+1)(s+2)/6;\quad f(-\frac{1}{3}) \approx P_3(-\frac{4}{3}) = 0.1745185$

(b) $P_1(s) = 0.2484244 + 0.2418235s$; $f(0.25) \approx P_1(-1.5) = -0.1143108$ $P_2(s) = P_1(s) - 0.04876419s(s+1)/2$; $f(0.25) \approx P_2(-1.5) = -0.1325973$
$P_3(s) = P_2(s) - 0.00283891s(s+1)(s+2)/6$; $f(0.25) \approx P_3(-1.5) = -0.1327748$

6. In the following equations, we have $s = (1/h)(x - x_0)$.

   (a) $P_1(s) = 4.48169 + 1.763410000s; P_1(0.43) = 2.224525200$
   $P_2(s) = P_1(s) + 0.3469250000s(s+1); P_2(0.43) = 2.348863120$
   $P_3(s) = P_2(s) + 0.04550166667s(s+1)(s+2); P_3(0.43) = 2.360604734$

   (b) $P_1(s) = 1.2943767 + 0.1957644000s; P_1(0.25) = 1.196494500$
   $P_2(s) = P_1(s) + 0.02758609500s(s+1); P_2(0.25) = 1.189597976$
   $P_3(s) = P_2(s) + 0.001767545000s(s+1)(s+2); P_3(0.25) = 1.188935147$

7. (a) $P_3(x) = 5.3 - 33(x+0.1) + 129.8\overline{3}(x+0.1)x - 556.\overline{6}(x+0.1)x(x-0.2)$

   (b) $P_4(x) = P_3(c) + 2730.243387(x+0.1)x(x-0.2)(x-0.3)$

8. (a) $P_4(x) = -6 + 1.05170x + 0.57250x(x-0.1) + 0.21500x(x-0.1)(x-0.3) + 0.063016x(x-0.1)(x-0.3)(x-0.6)$

   (b) Add $0.014159x(x-0.1)(x-0.3)(x-0.6)(x-1)$ to the answer in part (a).

9. (a) $f(0.05) \approx 1.05126$ (b) $f(0.65) \approx 1.91555$ (c) $f(0.43) \approx 1.53725$

10. $\Delta^3 f(x_0) = -6$, $\Delta^4 f(x_0) = \Delta^5 f(x_0) = 0$, so the interpolating polynomial has degree 3.

11. (a) $P(-2) = Q(-2) = -1$, $P(-1) = Q(-1) = 3$, $P(0) = Q(0) = 1$, $P(1) = Q(1) = -1$, $P(2) = Q(2) = 3$

    (b) The format of the polynomial is not unique. If $P(x)$ and $Q(x)$ are expanded, they are identical. There is only one interpolating polynomial if the degree is less than or equal to four for the given data. However, it can be expressed in various ways depending on the application.

12. $\Delta^2 P(10) = 1140$

13. The coefficient of x^2 is 3.5.

14. The coefficient of x^3 is $-11/12$.

15. The approximation to $f(0.3)$ should be increased by 5.9375.

16. $f(0.75) = 10$

17. $f[x_0] = f(x_0) = 1$, $f[x_1] = f(x_1) = 3$, $f[x_0, x_1] = 5$

18. (a) $P(1930) = 169,649,000$, $P(1965) = 191,767,000$, $P(2010) = 171,351,000$

    (b) The 1965 figure may not be very accurate, but the 2010 figure is likely to be extremely inaccurate.

19. Since $f[x_2] = f[x_0] + f[x_0, x_1](x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$,

$$a_2 = \frac{f[x_2] - f[x_0]}{(x_2 - x_0)(x_2 - x_1)} - \frac{f[x_0, x_1]}{(x_2 - x_1)}.$$

This simplifies to $f[x_0, x_1, x_2]$.

20. Theorem 3.3 gives

$$f(x) = P_n(x) + \frac{f^{n+1}(\xi(x))}{(n+1)!}(x - x_0)\dots(x - x_n).$$

Let $x_{n+1} = x$. The interpolation polynomial of degree $n+1$ on $x_0, x_1, \dots, x_{n+1}$ is

$$P_{n+1}(t) = P_n(t) + f[x_0, x_1, ..., x_n, x_{n+1}](t - x_0)(t - x_1)\dots(t - x_n).$$

Since $f(x) = P_{n+1}(x)$, we have

$$P_n(x) + \frac{f^{n+1}(\xi(x))}{(n+1)!}(x - x_0)\dots(x - x_n) = P_n(x) + f[x_0, ..., x_n, x](x - x_0)\dots(x - x_n).$$

Thus,

$$f[x_0, ..., x_n, x] = \frac{f^{n+1}(\xi(x))}{(n+1)!}.$$

21. Let $\tilde{P}(x) = f[x_{i_0}] + \sum_{k=1}^{n} f[x_{i_0}, \dots, x_{i_k}](x - x_{i_0}) \cdots (x - x_{i_k})$ and $\hat{P}(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, \dots, x_k](x - x_0) \cdots (x - x_k)$. The polynomial $\tilde{P}(x)$ interpolates $f(x)$ at the nodes $x_{i_0}, \dots, x_{i_n}$, and the polynomial $\hat{P}(x)$ interpolates $f(x)$ at the nodes $x_0, \dots, x_n$. Since both sets of nodes are the same and the interpolating polynomial is unique, we have $\tilde{P}(x) = \hat{P}(x)$. The coefficient of x^n in $\tilde{P}(x)$ is $f[x_{i_0}, \dots, x_{i_n}]$, and the coefficient of x^n in $\hat{P}(x)$ is $f[x_0, \dots, x_n]$. Thus, $f[x_{i_0}, \dots, x_{i_n}] = f[x_0, \dots, x_n]$.

Exercise Set 3.3, page 135

1. The coefficients for the polynomials in divided-difference form are given in the following tables. For example, the polynomial in part (a) is

$$H_3(x) = 17.56492 + 3.116256(x - 8.3) + 0.05948(x - 8.3)^2 - 0.00202222(x - 8.3)^2(x - 8.6).$$

(a)	(b)	(c)	(d)
17.56492	0.22363362	−0.02475	−0.62049958
3.116256	2.1691753	0.751	3.5850208
0.05948	0.01558225	2.751	−2.1989182
−0.00202222	−3.2177925	1	−0.490447
		0	0.037205
		0	0.040475
			−0.0025277777
			0.0029629628