

hw 08

$$2. (i) P(x) = \underset{*}{f(x_0)} \underset{\dots}{L_{n,0}(x)} + \dots + \underset{*}{f(x_n)} \underset{\dots}{L_{n,n}(x)} \quad (3.1)$$

for each $0 \leq i \leq n$, we have one multiplication between $f(x_i)$ and $L_{n,i}(x)$

$$\text{For } L_{n,k}(x) = \frac{\underset{*}{(x-x_0)} \dots \underset{*}{(x-x_{k-1})} \dots \underset{*}{(x-x_{k+1})} \dots \underset{*}{(x-x_n)}}{\underset{*}{(x_k-x_0)} \dots \underset{*}{(x_k-x_{k-1})} \dots \underset{*}{(x_k-x_{k+1})} \dots \underset{*}{(x_k-x_n)}} \leftarrow * \quad (3.2)$$

There are $(n-1) \times 2 + 1 = 2n-1$ multiplications in $L_{n,k}(x)$, $0 \leq k \leq n$

Hence, for interpolation through evaluating (3.1) and (3.2),

we have $(2n-1) \times (n+1) + (n+1) = 2n(n+1)$ multiplications

(ii) In Algorithm 3.2, we have i multiplications for $F_{i,i}$, $1 \leq i \leq n$

$$\Rightarrow 1+2+3+\dots+n = \frac{n(n+1)}{2} \text{ multiplications}$$

$$\text{And } P_n(x) = \left(\dots \left(\underset{*}{a_n (x-x_{n-1})} + \underset{*}{a_{n-1}} \right) \underset{*}{(x-x_{n-2})} + \underset{*}{a_{n-2}} \right) \dots \underset{*}{(x-x_0)} + \underset{*}{a_0} \quad (3.5)$$

In (3.5), we have n multiplications

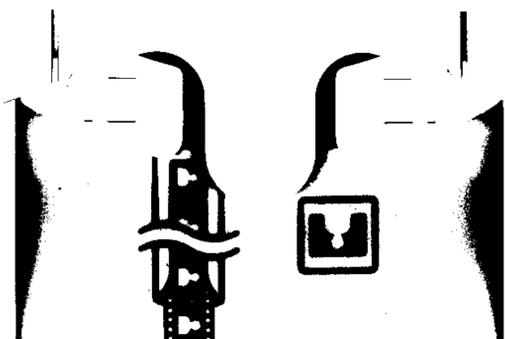
$$\text{Total: } \frac{n(n+1)}{2} + n = \frac{n(n+3)}{2} \text{ multiplications}$$

$$3. \text{ Let } f(x) = L_{3,0}(x) + L_{3,1}(x) + L_{3,2}(x) + L_{3,3}(x) - 1$$

Since $\deg(f) = 3$ and $f(x_0) = f(x_1) = f(x_2) = f(x_3) = 0$

$$\Rightarrow f(x) = 0$$

$$\Rightarrow L_{3,0}(x) + L_{3,1}(x) + L_{3,2}(x) + L_{3,3}(x) = 1, \text{ for all } x \in \mathbb{R}$$



$$4. (x_3, f(x_3)) = (0.5, 4) \Rightarrow f[x_2, x_3] = 10 \Rightarrow f[x_1, x_2, x_3] = 0 \Rightarrow f[x_0, x_1, x_2, x_3] = \frac{-100}{7}$$

b. (a) Let x_0, \dots, x_n be uniformly spaced nodes on $[a, b]$, $x_j = a + jh$, $h = \frac{b-a}{n}$

$\forall x \in [a, b]$, $x_{j-1} \leq x \leq x_j$ for some $1 \leq j \leq n$

$$\Rightarrow |x - x_{j-1}| \leq h^2 \quad \text{and} \quad \begin{cases} |x - x_{j+k}| \leq (k+1)h, & 1 \leq k \leq j-1 \\ |x - x_{j+t}| \leq (t+1)h \leq (t+j)h, & 1 \leq t \leq n-j \end{cases}$$

$$\Rightarrow |(x-x_0) \dots (x-x_{j-1})(x-x_{j+1}) \dots (x-x_n)| \leq |jh \cdot (j-1)h \cdot \dots \cdot 2h \cdot h^2 \cdot (j+1)h \cdot (j+2)h \cdot \dots \cdot nh|$$

$$= n! h^{n+1}, \quad \forall x \in [a, b]$$

(b)

$$|e^x - P_n(x)| = \left| \frac{e^{\xi_n}}{(n+1)!} (x-x_0)(x-x_1)\dots(x-x_n) \right|, \quad \text{where } \xi_n \in (a, b)$$

$$\leq \left| \frac{e^{\xi_n}}{(n+1)!} \right| \cdot h^{n+1} \quad (\text{By (a)})$$

$$\Rightarrow \lim_{n \rightarrow \infty} \max_{a \leq x \leq b} |e^x - P_n(x)| \leq \lim_{n \rightarrow \infty} \frac{e}{(n+1)!} \cdot h^{n+1} = 0$$

