Quiz 04

1\. (i) $X_0=-1,\ x_1=0,\ x_2=1$, to interpolate f and f' on x_0, x_1, x_2 with a single polynomial, the polynomial has to satisfy 6 conditions.

Hence, the lowest degree needed in general is 5

(ii)

$$H_5(X)=\sum_{j=0}^{2} P(x_j)H_{n,j}(x)+\sum_{j=0}^{2} P'(x_j)\hat{H}_{n,j}(x)$$

$$=[1-2(x-x_1)L'_{2,1}(x_1)]L^2_{2,1}(x)+2(x-x_1)L^2_{2,1}(x)$$

$$=(x+1)^2(x-1)^2+2x(x+1)^2(x-1)^2$$

3\. $S_0(1)=S_1(1) \Rightarrow 1+B+2-2=1 \Rightarrow B=0$

$$\Rightarrow f'(0)=S_0'(0)=0$$

$S_0'(1)=S_1'(1) \Rightarrow 4-6=b \Rightarrow b=-2$

$$\Rightarrow f'(2)=S_1'(2)=-2-8+21=11$$

4\.

Interpolated value at $x=0.1$: 0.9910784064 9

1\. (ii) Another method:

Since $P(x_0)=P'(x_0)=0$, $P(x_2)=P'(x_2)=0$ and $L_{2,1}(x)=\frac{(x-1)(x-(-1))}{(0-1)(0-(-1))}=-(x-1)(x+1)$

We can assume $P(x)=(ax+b)L^2_{2,1}(x)=(ax+b)(x^2-1)^2$

$$\Rightarrow P'(x)=a(x^2-1)^2+(ax+b)(4x)(x^2-1)$$

Per-Duet

$P(x_1)=1 \Rightarrow b=1$

$P'(x_1)=2 \Rightarrow a=2 \Rightarrow P(x)=(2x+1)(x^2-1)^2$

2.

Given $x_1, \cdots, x_n$, $f(x_1), \cdots, f(x_n)$, $f'(x_1), \cdots, f'(x_n)$

Version I

Step 1 For $i=1,2,\cdots,n$

set $z_{2i-1}=x_i$;

$z_{2i}=x_i$;

$Q_{2i-1,1}=f(x_i)$;

$Q_{2i,1}=f(x_i)$;

$Q_{2i,2}=f'(x_i)$;

If $i\neq 1$, set $Q_{2i-1,2}=\dfrac{Q_{2i-1,1}-Q_{2i-2,1}}{z_{2i-1}-z_{2i-2}}$;

end

Step 2 For $i=3,4,\cdots,2n$

for $j=3,4,\cdots,i$

set $Q_{i,j}=\dfrac{Q_{i,j-1}-Q_{i-1,j-1}}{z_i - z_{i-j+1}}$;

end

end

Step 3 output $Q_{1,1}, Q_{2,2}, \cdots, Q_{n,n}$

Version II

Step 1 For $i=1,2,\cdots,n$

set $z_{2i-1}=x_i$;

$z_{2i}=x_i$;

$h_{2i-1}=f(x_i)$;

$h_{2i}=f(x_i)$;

If $i\neq 1$, $h_{2i-1}=\dfrac{h_{2i-1}-h_{2i-2}}{z_{2i-1}-z_{2i-2}}$;

end

Step 2 For $i = 1, 2, \cdots, n$

  Set $h_{2i} = f'(x_i)$;

end

Step 3 For $i = 3, 4, \cdots, 2n$

  for $j = 2n, 2n-1, \cdots, i$

    Set $h_j = \dfrac{h_j - h_{j-1}}{z_j - z_{j-i+1}}$;

  end

end

output : h