

## Homework Assignment for Week 16

Assigned Dec 31, 2010.

1. Section 6.2: Problems 1(a), 3(a), 5(a), 31.
2. (corrected) Continue from last week's homework problem, show that if  $p < -1$ , and  $c_i$  is an arbitrary point in the interval  $[x_{i-1}, x_i]$ , then  $h \sum_{i=2}^n c_i^p - \int_h^1 x^p dx = O(h^{p+1})$  (which implies that  $h \sum_{i=2}^n c_i^p = O(h^{p+1})$ ,  $p < -1$  and  $h \sum_{i=1}^n x_{i-\frac{1}{2}}^q - \int_0^1 x^q dx = O(h^{q+1})$ ,  $-1 < q < 1$ ).

Hint: use monotonicity of the function  $x^p$ .

Remark: for general  $r \in R$ ,  $h \sum_{i=2}^n x_{i-\frac{1}{2}}^r - \int_h^1 x^r dx = O(h^{\min\{r+1, 2\}})$

3. Continue on previous problem, can you predict the order of convergence of Simpson's rule applied to  $\int_0^1 x^p dx$ ,  $-1 < p < 3$ ? Verify your answer numerically.