Homework Assignment for Week 15

Assigned Dec 21, 2010.

1. Section 4.9: Problems 4, 6, 7, 8.

2. Section 6.1: Problems 6, 15, 16.

3. Choose your favorite scheme to find the area enclosed by the curve $x^4 + y^2 = 1$ numerically. Try to find 10 correct digits with as little nodes as possible. Analyze your computed data at Differnent n and h and report the order of your scheme. You are encouraged to explore any methods that I mentioned in class, or your own method. You will find it very much rewarding.

4. Some of you have raised the question about the convergence of improper integrals. You can start with this problem, which is aimed to investigate what happens when you apply standard schemes to improper integrals. So do not use the techniques you learned in class, such as taking out the singular part or a change of variable. Just apply the scheme to the integrals directly.

   (a) Verify numerically that the midpoint rule $h\sum_{i=1}^{n} f(x_{i-\frac{1}{2}})$ indeed converges to $\int_0^1 f(x)dx$ as $h \to 0$, where $f(x) = x^{\frac{1}{2}}$, $h = 1/n$ and $x_{i-\frac{1}{2}} = (i - \frac{1}{2})h$. Can you find the order of convergence numerically?

   (b) Do the same for $f(x) = x^{-\frac{1}{2}}$, which is integrable but not bounded on $(0, 1)$ and the usual convergence Theorem for Riemann sum does not apply. Nevertheless, it still converges.

   (c) Now compare $h\sum_{i=2}^{n} f(x_{i-\frac{1}{2}})$ and $\int_h^1 f(x)dx$ with $f(x) = x^{-\frac{3}{2}}$. Note that now the integral $\int_h^1 f(x)dx$ will diverge as $h \to 0$. Surely you will expect the same for the sum. Show numerically that the difference between the sum and the integral is of lower order. Can you guess the order before computing?

   (d) I will formulate the above problem next week as homework problem, in a way that you can prove your numerical result rigorously. It will be easier for you to learn if you try it first numerically. Enjoy it.