# Homework Assignment for Week 08

Assigned Nov 04, 2010.

1. Implement Algorithm 3.2 (or its compact version using $F_i$ instead of $F_{i,k}$) as a function, with the data $x = (x_0, \cdots, x_n)$, $y = (f(x_0), \cdots, f(x_n))$ as input and the divided differences as output. Be careful that, in matlab, all indices starts with 1, not 0.

   It should be quite easy to find an existing code from the internet, but you are required to write this one from scratch since it is a simple and compact algorithm, very suitable for quiz 3 and midterm 2.

   A sample code for evaluation of (3.5) through nested expression (after the divided differences are calculated) is provided on the course homepage.

2. How many multiplications are needed for interpolation through evaluating (3.1) and (3.2) directly? How many multiplications are needed for interpolation through Algorithm 3.2 together with evaluation of (3.5) via nested expression?

3. Show that if $x_0$, $x_1$, $x_2$, and $x_3$ are distinct, then $L_{3,0}(x)+L_{3,1}(x)+L_{3,2}(x)+L_{3,3}(x) = 1$ for all $x \in \mathbb{R}$.

4. Section 3.2: Problems 17. Then add another data point $(x_3, f(x_3)) = (0.5, 4)$. What is $f[x_0, x_1, x_2, x_3]$?

5. Section 3.2: Problems 16, 19, 21.

   Hint for problem 16: similar to problem 17.

6. Let $x_0, \cdots, x_n$ be uniformly spaced nodes on $[a, b]$ with $x_j = a + jh$, $h = (b - a)/n$.

   (a) Show that $|(x - x_0) \cdots (x - x_n)| \le n! h^{n+1}$ on $a \le x \le b$.

   (b) Let $P_n$ be the degree $n$ interpolating polynomial of $e^x$ with uniformly spaced nodes on $[0, 1]$. Show that

   $$\max_{0 \le x \le 1} |e^x - P_n(x)| \to 0 \quad \text{as } n \to \infty$$

   Note that uniform convergence of interpolating polynomials as in (b) does not hold in most cases. Try estimating the interpolation error of $1/(1 + x^2)$ on $[-5, 5]$ with uniformly spaced nodes and you'll see why.

7. Skip the materials from page 123, after (3.11) till end of section 3.2.