Homework Assignment for Week 06

Assigned Oct 22, 2010.

1. Section 2.5: Problems 14, 15, 16.

   Hint for problem 16: Show that

$$\frac{\hat{p}_n - p}{p_n - p} = \frac{\lambda(\delta_n + \delta_{n+1}) - 2\delta_n + \delta_n\delta_{n+1} - 2\delta_n(\lambda - 1) - \delta_n^2}{(\lambda - 1)^2 + \lambda(\delta_n + \delta_{n+1}) - 2\delta_n + \delta_n\delta_{n+1}}$$

2. Find out $\hat{p}_n$ explicitly for Aitken's Δ^2 method for the following sequence: $a_n = 1/n$, $b_n = 1/n^2$, $c_n = \alpha^n$, $0 < \alpha < 1$ and $d_n = 2^{-2^n}$. Note the first two are linearly convergent sequence but does not satisfy the assumption of Theorem 2.13. The last one is a quadratically convergent sequence. Are the convergence accelerated in these cases?

3. On page 91, Horner's algorithm, the comments in the parenthesis 'Compute b_{j-1} for Q' may be a little confusing. We recast it as follows:

   We have $Q(x) = b_n x^{n-1} + \cdots + b_1$, therefore $Q(\bar{x})$ can be evaluated through the nested expression:

$$Q(\bar{x}) = (\cdots((b_n\bar{x} + b_{n-1})\bar{x} + b_{n-2})\bar{x} + \cdots) + b_1$$

   Define $c_n = b_n (= a_n)$ and for $j = n-1, n-2, \cdots, 1$, let $c_j = c_{j+1}\bar{x} + b_j$. Obviously, $c_1 = Q(\bar{x}) = P'(\bar{x})$.

   How should the comment 'Compute b_{j-1} for Q' be changed in terms of the notation c_j in Horner's algorithm on page 91?

4. Section 2.6: Regarding Problems 1 and 2.

   Due to time constraint, it is unlikely for you to implement Horner's algorithm to solve these problems before midterm 01. So instead we just study them theoretically.

   Suppose that you are given a subroutine horner_deg_4.m, which takes the input $a_0, \cdots, a_4, \bar{x}$ and outputs $P(\bar{x})$ and $P'(\bar{x})$ using Horner's algorithm, where $P(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.

   (a) Give an operation count for each Newton's iteration $x_{k+1} = x_k - P(x_k)/P'(x_k)$, where $P(x_k)$ and $P'(x_k)$ are obtained by calling horner_deg_4.m.

   (b) Suppose that x^* is a convergent root of $P(x)$, how do you modify horner_deg_4.m to obtain the deflated polynomial $Q(x) = P(x)/(x - x^*)$ without extra cost?

   A subroutine horner_deg_4.m will be posted on course homepage later.

5. Suppose Müller's method is applied to locate a root of $f(x) = x^3 - 2 = 0$, with $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$. What is x_3? What happens if you apply Müller's method to solve a quadratic equation $ax^2 + bx + c = 0$?

6. Show that secant method applied to solving $f(x) = e^x - 1 = 0$ with $0 < x_0 < x_1$ always converges to the solution $x^* = 0$.

   Hint: This f is convex and strictly increasing.

7. Show that the method of false position applied to solving $f(x) = e^x - 1 = 0$ with $x_0 > 0$, $x_1 < 0$ always converges to the solution $x^* = 0$.

8. (Harder, and hopefully enlightening) What is the order of convergence for previous problem with $x_0 = 1.25643$, $x_1 = -0.01$?

   Hint: Find a few x_n and see what happens. x_0 is a root of $e^x - 1 = 2x$, chosen so that you can see some clean numbers. It is not really important.