Assignment 2.

(1) Here is an alternative proof for the rank theorem.

(a) If a linear transformation L is nonsingular, and v_i are linear independent vectors, show that Lv_i are linearly independent.

(b) Use part (a) to show that

$$\text{rank}(PA) = \text{rank}(A)$$

if P is a nonsingular matrix.

(2) let $A \in R^{m \times n}$ and $B \in R^{n \times k}$.

(a) Show that $\text{rank}(AB) \leq \text{rank}(A)$.

(b) Is it true that $\text{rank}(AB) = \text{rank}(B)$? Explain.

(c) Show that $\text{rank}(AB) = \text{rank}(B)$ if and only if $\text{range}(B) \cap \ker|(A) = \{0\}$ where $\text{range}(B)$ is the linear span of columns of B.

(3) Let

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

Find scalars α and β such that $A^{10} = \alpha A + \beta I$.

(4) Let A be a $n \times n$ matrix with all its entries $a_{i,j} = 1$. Find the rank of $A - \lambda I$ as a function of λ. As a consequence, find one eigenvalue of A

(5) Let u_i, $i = 1, \cdots, m$ and v_j, $j = 1, \cdots, n$ be eigenvectors associated with eigenvalues λ_1 and λ_2 respectively with $\lambda_1 \neq \lambda_2$. Show that u_i's are linearly independent from v_j's.