Assignment 3.

Given Oct 13 2000, due Oct 20 2000.

Remark: You should pay extra attention about your proof. Try to write your proof in a rigorous fashion and have a classmate proofread it for you.

(1) Do exercises 6, 9, 13, 15-18, 20, 22-27 from Chap 5. of the textbook.

(2) (a) Find a proper (i.e. nontrivial) subspace of $\mathcal{N}$ in example 3 (page 42).
  (b) Find a finite set E such that $\mathcal{L}(E) = \mathcal{N}$

(3) Describe $\mathcal{L}(E)$ where $E = \{\mathbf{1}_\Omega \mid \Omega \subset (0,1]\}$ and give a proof (we already gave the answer in class). Is the function

$$f(x) = \begin{cases} 0 & \text{if} \quad x \in (0,1] \text{ is rational} \\ 1 & \text{if} \quad x \in (0,1] \text{ is irrational} \end{cases}$$

in $\mathcal{L}(E)$? How about

$$g(x) = 2^{-n} \quad \text{if} \quad x \in (\frac{1}{n+1}, \frac{1}{n}], n = 1, 2, 3, \cdots?$$