MATH 543
METHODS OF APPLIED MATHEMATICS I
Homework Set II

For October 15, 2009

QUESTIONS

1. 1. Prove that the Fourier coefficients of any $|f> \in L^2_w(a,b)$ form a Hilbert space. First prove that the space of such coefficients form an inner product space then prove that this inner product space is complete. This space is $\mathbf{l_2}$ (see set 1). Prove that l_2 and $L^2_w(a,b)$ are isomorphic. See page 196 of DK.
2. Prove that all finite dimensional inner product spaces are complete. See page 182-183 of DK.
3. Explain the importance of Bessel's inequality and Parseval's relation.
4. Assume that there exists an orthonormal basis $|e_i>$, $(i = 1, 2, \cdots)$ in $L^2_w(a,b)$. Then , for any $|f> \in L^2_{a,b}$, the sequence of vectors

$$|f_k> = \sum_{i=1}^{k} f^i \, |e_i>$$

with

$$f^i = < e_i | f >$$

has $|f>$ as the limit vector in the sense that

$$\lim_{k \to \infty} \rho(|f>, |f_k>) = 0$$

5. Find and prove the following for the Hermite Polynomials by using its generating function where $a_n = \frac{1}{n!}$:
(a) $H_n(-x) = (-1)^n \, H_n(x)$, **(b)** $||H_n||$, **(c)** $H_n(0)$
(d) $H_{n+1} - 2xH_n = 2nH_{n-1}$, **(e)** $H_n'' - 2xH_n' + 2nH_n = 0$,
(f) $\frac{d^m}{dx^m} H_n = 2^m \, \frac{n!}{(n-m)!} H_{n-m}$ (Use the Rodriguez formula to prove this property of the Hermite polynomial).

6. Prove that Orthogonal Polynomials $C_n(x)$ with $x \in [a,b]$ has n zeros in $[a,b]$.

7. Find $< xC_n, C_m >$ where $C_n(x)$ is any one of the classical orthogonal polynomial with $x \in [a,b]$

8. Express the function $f(x) = -1 \quad$ for $-1 \leq x < 0$ and $f(x) = 1$ for $0 < x \leq 1$ in terms of the Legendre polynomials. Find the first five terms and discuss the convergence of the series.