MATH 543
METHODS OF APPLIED MATHEMATICS I
Final Exam January 05, 2011
Monday 15.30-17.30., SAZ04

QUESTIONS: Solve any three out of four problems

[35]**(1)** The generating function of the legendre polynomials is given as

$$(t^2 - 2xt + 1)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$$

Find or prove the following for the Legendre polynomials by using its generating function

(a) $P_n(-x) = (-1)^n P_n(x)$,
(b) $\int_0^1 P_n(x)dx$,
(c) $P'_{n+1}(x) - P'_{n-1}(x) - (2n+1)P_n(x) = 0$,
(d) $P'_{n+1} - xP'_n - (n+1)P_n = 0$

[35]**(2)**. **a)** Let $C_n(x)$ be a classical orthogonal polynomial with the weight function $w(x)$ where $x \in [a, b]$. Let

$$I(m, n, k) = \int_a^b w(x)\, C_n(x)\, C_m(x)\, C_k(x)\, dx$$

For which values of the integers m, n and k the integral I vanishes. Find an expression of I.

b) Find the dominant term of the integral

$$\int_0^\pi \sqrt{t^2 + 1}\, e^{\lambda \sin t}\, dt$$

where $\lambda >> 1$.

[35]**(3)**. Solve the following problem by the of Green's function

$$y'' + y = f(x), \quad 0 < x < a,$$
$$y(0) = 0, \quad y(a) = 0 \tag{1}$$

Here $f(x)$ is a given function.

[35](**4**). Find a first order approximate solution of the following problem

$$y'' + y = \epsilon\, y\, (y')^2,$$
$$y(0) = 1, \quad y'(0) = 0,$$

where $\epsilon << 1$. Discuss the validity of the approximate solution.