MATH 543
METHODS OF APPLIED MATHEMATICS I
First Midterm Exam

November 1, 2010
Monday 13.40-15.30., SAZ02

QUESTIONS:

(1). (a). Give a definition of a basis, **(b)** state the Bessel's inequality, **(c)** State the theorem on Parseval's relation (Parseval's identity) and prove it.

(2). Prove that the Fourier coefficients of function in $L^2_w(a,b)$ form a Hilbert space. First prove the space of such coefficients form an inner product space then prove that this space is complete. This space is known as l_2.

(3). Let the classical orthogonal polynomials be $C_n(x)$, with $x \in [a,b]$ and weight function $w(x) > 0$ for all $x \in [a,b]$. Prove that C_n's have n number real distinct roots in $[a,b]$.

(4). The sequence $D_n(x) = \sqrt{\frac{n}{\pi}}\, e^{-nx^2}$, $(n = 1, 2, 3, \cdots)$. defines a distribution over $(-\infty, \infty)$, called the Dirac δ-function, $\delta(x)$. Prove that

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1,$$
$$\int_{-\infty}^{\infty} \delta(x)\, f(x)\, dx = f(0),$$

for all good functions $f(x)$.