**MATH 543**
**METHODS OF APPLIED MATHEMATICS I**
**Second Midterm Exam**

**December 5, 2003**
Friday 18.00-20.00, SAZ-18

**QUESTIONS** First three problems must be solved by the of **the method of Green's function**. Solve any two out of these three questions. Choose any two of the problems 4-7. So the total number of problems you should solve must be four.

**1**. Solve $u'' + u = f(x)$ with $u(0) = \gamma_1$ and $u'(0) = \gamma_2$ where $f(x)$ is a continuous function over $x \in I$ and $\gamma_1$ and $\gamma_2$ are some real constants.

**Solution**

$$u(x) = \int_0^x \sin(x-y)\, f(y)dy + \gamma_1 \cos x + \gamma_2 \sin x$$

**2**. Solve $u'' + 6u' + 9u = f(x)$ with $u(0) = 0$ and $u'(0) = 0$

**Solution**:

$$u(x) = \int_0^x (x-y)\, e^{3(y-x)}\, f(y)\, dy$$

**3**. Solve $u''' + u'' = f(x)$ with $u(0) = u'(0) = u''(0) = 0$

**Solution**:

$$u(x) = \int_0^x [x - y - 1 + e^{y-x}]\, f(y)\, dy$$

**4**. Let $u'' + p(z)u' + q(z)u = 0$. Let $z = z_0$ be a regular singular point of this DE. Let the difference $r_1 - r_2 = 2N$ of the indices $r_1$ and $r_2$ where $N$ is a non-negative integer ($r_2 > r_2$). Prove that the solution corresponding to the second index $r = r_2$ has logarithmic singularity at $z = z_0$.
**5**. Prove that any second order differential equation having only two regular singular points are transformable to a DE with constant coefficients (or to a Eulerian type of second order DE.

**6**. Solve $z^2u'' + (z^2 + z)u' - u = 0$ about $z = 0$.

**Solution**: $u(z) = Au_1(z) + Bu_2(z)$ where $A$ and $B$ are some constants and

$$u_1(z) = \sum_{n=0} \frac{(-1)^n \, z^{n+1}}{(n+2)!} = \frac{1}{z}\,(e^{-z} - 1) + 1, \quad u_2(z) = 1 - \frac{1}{z}$$

**7**. Prove that the solution of a second order DE about an ordinary point exists and unique.