MATH 543
METHODS OF APPLIED MATHEMATICS I
First Midterm Exam

October 30, 2003

Thursday 18.00-20.00, SAZ-18

QUESTIONS

1. Prove the following: If $(u_1, u_2, \cdots, u_n)$ is a finite orthogonal family of points in an inner product space X and $x \in X$, then $||x - \sum_{k=1}^{n} \alpha^k \, u_k||$ has its minimum value when $\alpha^k =< x, u_k >$ and

$$\sum_{k=1}^{n} | < x, u_k > |^2 \leq ||x||^2$$

where $<,>$ is the inner product defined on X and $||x||$ is the norm defined through the inner product $< x, x >$

2. Prove that the set of orthonormal vectors $|e_i >$ $(i = 1, 2, ...)$ form a basis of $L^2_w(a, b)$ if and only if the Fourier coefficients with respect to the basis $|e_i >$ satisfy Parseval's relation

3. The generating function of the Legendre polynomials is given by

$$\frac{1}{\sqrt{t^2 - 2xt + 1}} = \sum_{n=0}^{\infty} t^n \, P_n(x), \quad |t| < 1$$

By the use of this generating function

(a) prove that $P'_{n+1}(x) - x \, P'_n(x) - (n + 1)P_n(x) = 0$,

(b) find $\int_0^1 P_n(x)$ dx.

4. Prove that classical orthogonal polynomials $C_n(x)$ with $x \in I = [a, b]$ has n distinct zeros in I

5. Prove the following:

(a)

$$\int_{-\epsilon}^{\epsilon} \delta(x - c)dx = \begin{cases} 1 & c \in (-\epsilon, \epsilon) \\ 0 & c \notin (-\epsilon, \epsilon) \end{cases}$$

(b)

$$\int_{-\infty}^{\infty} \delta(x^2 - a^2)dx = \frac{1}{2|a|}[\delta(x - a) + \delta(x + a)]$$