Homework Assignment for Week 03

Assigned Mar 09, 2011.

1. Verify the duality formula of the Green's function for the differential equation

$$y'' + p(x)y' + q(x)y = f(x), \quad y(0) = 0, \ y(1) = 0$$

and its adjoint equation

$$z'' - p(x)z' + q(x)y = h(x), \quad z(0) = 0, \ z(1) = 0$$

In other words, verify that $g_\xi(x) = g_x^*(\xi)$ by considering the integral $\int_0^1 y(x)h(x)dx$.

A corollary is $g_\xi(x) = g_x(\xi)$ if $p \equiv 0$. In this case, the differential operator is self-adjoint.

2. With our preliminary analysis in class, can you give a rigorous error estimate for the example given in section 2.1.3?