§4.2 Selection

Changes in gene frequency

One of the conclusions of the previous section was that the existence of selective differences among genotypes generally leads to changes in gene frequencies. Assuming Hardy-Weinberg proportions, the frequency $p'$ of $A_i$ in any generation is related to its frequency $p$ in the previous generation by

$$p' = \frac{w_{11}p^2 + w_{12}pq}{w_{11}p^2 + 2w_{12}pq + w_{22}q^2} = f(p) \quad (2.1)$$

where the $w_{ij}$ are defined in Section 4.1. The change $\Delta p$ in the frequency of $A_i$ thus

$$\Delta p = pq \frac{w_{11}p + w_{12}(1-2p) - w_{22}q}{w_{11}p^2 + 2w_{12}pq + w_{22}q^2}. \quad (2.2)$$

Clearly $\Delta p = 0$ whenever $p = 0$ or $p = 1$, corresponding to fixation of $A_2$ or $A_1$. $\Delta p$ is also zero when

$$p = p^* = \frac{w_{12} - w_{22}}{(w_{12} - w_{22}) + (w_{12} - w_{11})}. \quad (2.3)$$

The evolution of the process is obtained by iterating the transformation law $(2.1)$. The following classical results are readily established (cf. Figure 2.1 below) independent of the initial $p$ ($0 < p < 1$).

$$\lim_{n \to \infty} f^n(p) = \lim_{n \to \infty} f(f^{(n-1)}(p)) = 1 \quad (=0) \text{ when } w_{11} \geq w_{12} > w_{22} \quad (w_{22} \geq w_{12} > w_{11}),$$

$$\lim_{n \to \infty} p_n = p^* = \frac{w_{12} - w_{22}}{w_{12} - w_{11} - w_{22}} \quad \text{when } w_{12} > \max(w_{11}, w_{22}).$$

In the case $\min(w_{11}, w_{22}) > w_{12}$

$$\lim_{n \to \infty} p_n = 1 \quad \text{for } p > p^*, \quad 0 \text{ for } p < p^*.$$

Figure 2.1 shows what happens to $f_{(n)}(p)$ in graphical form. The rigorous details are easily supplied.
The equilibrium $p^*$ is of great importance biologically because it entails the simultaneous existence at an equilibrium involving all genotypes. Thus when the heterozygote is the most fit of the three genotypes a stable polymorphism (with all forms) will be maintained. The model of heterozygote advantage (also called the principle of overdominance) has been central to the development of theories on the existence of genetic variability.

Perhaps the best-known example of this situation in man concerns the phenomenon of sickle-cell anaemia. The maintenance of high frequencies for both the sickle cell gene and its normal allele in certain East African tribes appears to be due to a selective advantage of heterozygotes brought about by the increased resistance of such heterozygotes to malaria.

Figure 4.2.1

\[ w_{12} > w_{11}, \quad w_{22} \]

\[ w_{12} < w_{11}, \quad w_{22} \]