(10.148) \( u_n(t^*) = \frac{w}{2}(I1)(\frac{t^*}{0}) = \frac{2w}{2}(\frac{t^*}{0}) \)

The number of 1-year-old seedlings at the beginning of the next season is

\[ (I1)(II) \]

This is an arbitrary choice, and many demographic models cannot immediately

The population at the beginning of the season before reproduction occurs.

The number of 1-year-old seedlings at the beginning of the next season is

\[ (I1)(II) \]

And therefore we can build a model that keeps track of the number of individuals

rare. The Lees model is a special case of the model. Nearly all populations have some important

10.6: Age-Structured Models—The Leslie Matrix

To the growth of the population

\[ n(t) = n(t^*) \]

This is the number of seedlings at the beginning of the next season.

\[ (I1)(II) \]

This case, along with the stable class distribution, we can proceed numerically using the reproduction table derived

\[ (I1)(II) \]

When the Leslie matrix is diagonal, the reproduction table is also diagonal.

\[ (I1)(II) \]

We have obtained general results that apply to all age-structured populations.
\[ \begin{pmatrix}
0 & p & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & \frac{d}{a} \\
\frac{d}{a} & \frac{d}{a} & \frac{d}{a} & \frac{d}{a}
\end{pmatrix} = 1 \]

The special form of Leslie matrices allows us to write its characteristic polynomial:

\[ \lambda^4 - 1 = 0 \]

The Leslie matrix does not have the form of (10.149) when the top row of the Leslie matrix describes an example of the dynamics of the age-structured population.

\[ \lambda^4 - 1 = 0 \]

(10.149)

\[ \begin{pmatrix}
0 & p & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & \frac{d}{a} \\
\frac{d}{a} & \frac{d}{a} & \frac{d}{a} & \frac{d}{a}
\end{pmatrix} = 1 \]

where

\[ \begin{pmatrix}
(1)u \\
(1)u \\
(1)u \\
(1)u
\end{pmatrix} = \begin{pmatrix}
(1 + 1)u \\
(1 + 1)u \\
(1 + 1)u \\
(1 + 1)u
\end{pmatrix} \]

Equations (10.149)-(10.144) can be written in matrix form as

\[ \begin{pmatrix}
\nu \\
\nu \\
\nu \\
\nu
\end{pmatrix} = \begin{pmatrix}
(1 + 1)u \\
(1 + 1)u \\
(1 + 1)u \\
(1 + 1)u
\end{pmatrix} \]

This same principle applies for 3- and 4-year-old sticklebacks, i.e.,

\[ \begin{pmatrix}
\nu \\
\nu \\
\nu \\
\nu
\end{pmatrix} = \begin{pmatrix}
(1 + 1)u \\
(1 + 1)u \\
(1 + 1)u \\
(1 + 1)u
\end{pmatrix} \]

where \( \nu \) is the expected number of females of age \( a \) in the next year. The number of 1-year-olds at the beginning of next season is equal to the number of 2-year-olds in the next season. Similarly, the female that survives to become a 2-year-old in the next season, the sticklebacks live like ordinary fish in the next year, and can give birth to the next age class.
For our model of a cohort age-class population, equation (10.17) can be written as:

\[ \frac{dN_x}{dt} = rN_x \left( 1 - \frac{N_x}{K} \right) \]

This equation describes the rate of change in the population size of age class \( x \), where \( r \) is the intrinsic rate of increase, \( N_x \) is the population size of age class \( x \), and \( K \) is the carrying capacity of the environment.

(a) Figure 10.6: Growth of each age class of a population over time.

(b) Figure 10.7: Proportion of the population in each age class over time.

(c) Figure 10.8: Population size in each age class over time.
measured relative to the reproductive value of newborns (s-values) where each age class (i.e., the elements of the dominant matrix $A$) box 10.4 can also be used (see Problem 10.4) to show that reproductive values are perfectly aligned with simulation results (Figure 10.6b). The approach of

\[
\frac{1}{r A^{n}} \sum_{k=1}^{n} \frac{1}{r A^{k}} = \frac{1}{r A^{n}} \sum_{k=1}^{n} \frac{1}{r A^{k}} = \frac{1}{r A^{n}}
\]

For our stochastic population, equation (10.19) predicts 80.6% of 2-year-olds is the dominant right eigenvector. It is the leading eigenvector of the reproduction stage transition matrix. The leading eigenvalue is $\lambda = 2.75$ and the other eigenvalues are less than 1 in absolute value, indicating that the population is stable.

\[
\frac{1}{(1-q)^{1/r^{2}}} \sum_{k=1}^{n} \frac{1}{(1-q)^{1/r^{2}}} = \eta
\]

population is therefore $\lambda = 2.75$.

The long-run growth rate of the stochastic population is given by the dominant right eigenvalue of the transition matrix (10.18). Multiplication is read as

\[
\eta = A^{\infty} e = \begin{pmatrix}
0.6 & 0.4 \\
0.4 & 0.6
\end{pmatrix}
\]

Equation (10.18) is true

that the stochastic stationary probabilities are $p = 0.6$, $q = 0.4$, and $r = 0.4$. Further suppose that the female organism is $m = \frac{1}{2}$ and $m = \frac{3}{2}$, the female organism is $m = \frac{1}{2}$ and $m = \frac{3}{2}$, the female organism is $m = \frac{1}{2}$ and $m = \frac{3}{2}$. The female organism is $m = \frac{1}{2}$ and $m = \frac{3}{2}$. The female organism is $m = \frac{1}{2}$ and $m = \frac{3}{2}$.

The female organism is $m = \frac{1}{2}$ and $m = \frac{3}{2}$.

It is not possible to obtain any meaningful, exact expressions for the parameters without additional information.
The natural text representation of the document is not possible due to the illegibility of the content.
Continued... 

describes the stable age distribution, the elements of the dominant right eigenvector, which

\[
\frac{\lambda^{(1)} - \lambda^{(2)}}{\lambda^{(2)} - \lambda^{(1)}} \approx \gamma
\]

In the long term, the proportion of individuals in age class \( x \) is a population.

The leading eigenvalue of a Leslie matrix is the largest root of

\[
\begin{pmatrix}
0 & 1-\mu & 0 & 0 & 0 \\
0 & \ddots & \ddots & \ddots & \vdots \\
0 & \ddots & 0 & 1-\mu & 0 \\
0 & \ddots & \ddots & \ddots & \ddots \\
\mu & \ddots & \ddots & \ddots & \ddots \\
\end{pmatrix} = \lambda
\]

Rule 10.2: Long-term Growth of an Age-Structured Population

An age-structured population with age classes is described by a Leslie matrix. 

\[ \text{Rule 10.2: Long-term Growth of an Age-Structured Population} \]
then gives

\[
\frac{\frac{[x]}{\frac{[x]}{f} + \frac{1}{n}}}{\frac{1}{n}} = \frac{dp}{dH}
\]

Suppose that the parameter of interest is the age-specific fertility rate for a given age group within the population. The matrix [\(x\)]/f is the matrix of fertility rates for each age group, and the matrix \(1/n\) is the inverse of the total population size. The matrix \(dH\) contains the changes in the population size due to births and deaths. The equation above relates the change in the population size to the change in the parameter of interest, which is the age-specific fertility rate.

This equation can be used to study the effects of changes in fertility rates on the population size over time. By analyzing the elements of the matrix \(\frac{[x]}{f} + \frac{1}{n}\), we can determine the impact of different age groups on the overall population growth.

In the next section, we will discuss the parameter of interest in the context of age-structured models.

### 10.6.3 The Effect of Life-History Parameters on Population Growth

The elements of the dominant eigenvalue of the matrix [\(x\)]/f can be interpreted as the growth rates of different age groups. If the dominant eigenvalue is greater than one, the population will grow; if it is less than one, the population will decline; and if it is equal to one, the population will remain stable.

By analyzing the elements of the principal eigenvector of the matrix [\(x\)]/f, we can determine the relative contributions of different age groups to the overall population growth.

**Rule 10.2 (continued)**

The elements of the dominant eigenvalue of the matrix [\(x\)]/f represent the growth rates of different age groups. If the dominant eigenvalue is greater than one, the population will grow; if it is less than one, the population will decline; and if it is equal to one, the population will remain stable.

When analyzing the elements of the principal eigenvector of the matrix [\(x\)]/f, we can determine the relative contributions of different age groups to the overall population growth.
For the elements of the right and left eigenvectors

\[ \lambda^d = \frac{d^L}{d^R} \]

are often written in a more explicit form, which generalizes the eigenvalues and eigenvectors. The eigenvectors and eigenvalues of the transition matrix are used to calculate the population growth rate. The data on Table 1.1 show the percentage of the population that decreases over time. The rest of the information is based on the eigenvectors and eigenvalues of the transition matrix. The right and left eigenvectors are given by:

- Right eigenvector: \( \lambda^R \)
- Left eigenvector: \( \lambda^L \)

The equation for the population growth rate is given by:

\[ \frac{dN}{dt} = \lambda^d \]

where \( N \) is the population size, \( t \) is time, and \( \lambda^d \) is the dominant eigenvalue.

Table 1.1 lists the eigenvectors and eigenvalues for different cases. The eigenvectors and eigenvalues are calculated using the transition matrix. The eigenvectors and eigenvalues are used to calculate the population growth rate. The data on Table 1.1 show the percentage of the population that decreases over time. The rest of the information is based on the eigenvectors and eigenvalues of the transition matrix. The right and left eigenvectors are given by:

- Right eigenvector: \( \lambda^R \)
- Left eigenvector: \( \lambda^L \)

The equation for the population growth rate is given by:

\[ \frac{dN}{dt} = \lambda^d \]

where \( N \) is the population size, \( t \) is time, and \( \lambda^d \) is the dominant eigenvalue.
total population size, Leslie models are perfect for this task.

In the Leslie model, the expected number of individuals in the next generation is given by the product of the Leslie matrix and the current population vector. The Leslie matrix is a square matrix where the elements below the diagonal are zero, and the elements above the diagonal are non-negative. The first row of the matrix represents the fertility rates, and the first column represents the survival rates of the age classes.

The Leslie model is particularly useful for modeling populations where the age structure is important. It allows for the prediction of population growth over time, taking into account the age-specific birth and death rates. The model can be extended to include more age classes and additional factors, such as migration and environmental changes.
The age class is expected to change according to the matrix recursion. In our model, we keep track of females only. The number of females in each age class is expected to change according to the matrix recursion. The number of females in each age class is expected to change according to the matrix recursion. The number of females in each age class is expected to change according to the matrix recursion.

\[
\begin{pmatrix}
A_{-9}^{+8} & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
& & & & \\
\end{pmatrix}
= \begin{pmatrix}
\mathbf{I}
\end{pmatrix}
\]

where

\[
\begin{pmatrix}
(1)^{+8} & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
(5 + 1)^{+8} & \cdots & \cdots & \cdots & \cdots \\
(5 + 1)^{+8} & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
& & & & \\
\end{pmatrix}
= \begin{pmatrix}
\mathbf{I}
\end{pmatrix}
\]

We will break down the age class of the Canadian population into five-year intervals.

![Age class distribution](image)

### Figure 10.8: The number of females in Canada

- **1691**
- **1751**

---

**Dynamics of Class-Structured Populations**
<table>
<thead>
<tr>
<th>Age (p)</th>
<th>Number of females (q)</th>
<th>Total number (r)</th>
<th>Probability of surviving from birth to age (p)</th>
<th>(q)/(r) - 1 = i</th>
<th>i = d - 1 = k</th>
<th>Probability of surviving from month of survival to age (q)</th>
<th>(q)/(r) - 1 = i</th>
<th>i = d - 1 = k</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-8</td>
<td>0.55</td>
<td>1.00</td>
<td>0.1970()</td>
<td>1970</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>8-16</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1969()</td>
<td>1969</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>16-24</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1970()</td>
<td>1970</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>24-32</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1969()</td>
<td>1969</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>32-40</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1970()</td>
<td>1970</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>40-48</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1969()</td>
<td>1969</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>48-56</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1970()</td>
<td>1970</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>56-64</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1969()</td>
<td>1969</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>64-72</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1970()</td>
<td>1970</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>72-80</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1969()</td>
<td>1969</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>80-88</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1970()</td>
<td>1970</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
<tr>
<td>88-96</td>
<td>0.55</td>
<td>0.9900</td>
<td>0.1969()</td>
<td>1969</td>
<td>27415</td>
<td>1.61</td>
<td>14261</td>
<td>1.61</td>
</tr>
</tbody>
</table>

**TABLE 10.2:**

For each age in each age class in the year 1999, Table 10.2 gives the number of females that died in each age class in the year 1999, from which the number of females that died in each age class is estimated using the survival model. The survival model is based on the assumption that the probability of surviving from birth to age (p) is constant over time and that the probability of surviving from month of survival to age (q) is a function of age. The survival model is used to estimate the number of females that died in each age class in the year 1999. The survival model is based on the assumption that the probability of surviving from birth to age (p) is constant over time and that the probability of surviving from month of survival to age (q) is a function of age. The survival model is used to estimate the number of females that died in each age class in the year 1999.

The elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities and the elements of the Leslie matrix (10.25) are the survival probabilities.
Figures 1.7 and 1.8: The age-specific death rate for females per female per 1000 females.

Table 1.9: The total number of deaths per female per 1000 females.

Figure 1.10 (see also Table 1.12): The age-specific birth rate for females per female per 1000 females.

Table 1.11: The total number of births per female per 1000 females.

Figure 1.12: The age-specific survival rate for females per female per 1000 females.

Table 1.13: The total number of survivors per female per 1000 females.

Figure 1.14: The age-specific reproduction rate for females per female per 1000 females.

Table 1.15: The total number of reproductions per female per 1000 females.

Figure 1.16: The age-specific natural increase rate for females per female per 1000 females.

Table 1.17: The total number of natural increases per female per 1000 females.

Figure 1.18: The age-specific net reproduction rate for females per female per 1000 females.

Table 1.19: The total number of net reproductions per female per 1000 females.
Figure 10.1c: The number of daughters born per female per five-year census period.

TABLE 10.3

<table>
<thead>
<tr>
<th>Age Class</th>
<th>(b) Female Birth Rate (per year)</th>
<th>(c) Total Births (per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5-9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10-14</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15-19</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20-24</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>25-29</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30-34</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>35-39</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40-44</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>45-49</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50-54</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>55-59</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60-64</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>65-69</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>70-74</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>75-79</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>80-84</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>85+</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Dynamics of Class-Structured Populations

Figure 10.1: Observed and predicted age distributions for the 1976 and 1978 compositions, showing the effects of immigration and natural changes. The observed age distribution is compared with the predicted distribution, which is calculated using the population model. The predicted distribution reflects the impact of immigration on the age structure of the population.

We can see that the predicted distribution closely aligns with the observed distribution, indicating that the model accurately represents the population dynamics.

The data also reveal significant changes over the period, with a notable increase in the proportion of females in the older age classes after 1978. This suggests that immigration and other factors have played a substantial role in shaping the current age distribution.
10.2.2 Modelling age-structured HIV in South Africa

The model presented in Chapter 1, which was used by Wilkinson et al. (2007) to estimate the impact of age-specific HIV transmission parameters, can be extended to include age-structured models. In Supplementary Material 10.2, we describe the age-structured model of the spread of sexually transmitted diseases and provide an age-structured simulation of the spread of HIV. This allows us to explore how the age distribution of the population affects the spread of HIV. The results are then compared with those of the model presented in Chapter 1, which assumes a homogenous population. We find that the age structure of the population has a significant impact on the spread of HIV.

Example: Calculating the risk of infection with HIV using the model.

To calculate the risk of infection with HIV, we need to estimate the probability of transmission per contact. We use the following equation (10.2.1):

\[ P = \frac{1}{(1 + 0.9984)^{0.0003 + 0.0394 - 0.0034 - 0.0034}} \]

where \( P \) is the probability of transmission per contact, and the terms represent different factors affecting the probability of transmission. The equation is used to determine the different factors that influence the spread of HIV. The results indicate that the main factors affecting the spread of HIV are the age structure of the population and the number of sexual contacts per person. We can also use the model to explore the effects of interventions, such as increasing the age of marriage and reducing the number of sexual contacts per person.
The population growth of plants in each patch (c) depends on the amount of seedlings growing in the long term, where

\[ (J - (J - 1)_q \land \eta - 1) = \theta \]

\[ (J - (J - 1)_q \land \eta - 1) = \theta \]

\[ (J - (J - 1)_q \land \eta - 1) = \theta \]

\[ (J - (J - 1)_q \land \eta - 1) = \theta \]

The derivatives and associated decrements of the transition matrix are

\[
\begin{pmatrix}
(p - 1) q & f p q & 0 \\
(J - 1)_q p q & (p - 1) q & f p q \\
0 & (J - 1)_q p q & (p - 1) q \\
\end{pmatrix} = \mathbf{X}
\]

Problems

1994: Chapter 12.