Show that $\lim_{x \to 2} x^2 = 4$

Pf: we know that $|x^2 - 4| = |x-2||x+2|$

so if we choose $|x-2| < \frac{1}{3}$, then

$$|x+2| \leq |x-2| + |4| < 4\tfrac{1}{3}$$

$$\Rightarrow |x^2-4| = |x-2||x+2| < 4\tfrac{1}{3}\,|x-2|$$

$\forall$ given $\varepsilon > 0$, let $\delta = \min\left\{ \frac{1}{3}, \frac{\varepsilon}{4\frac{1}{3}} \right\}$

then, if $|x-2| < \delta$

$$\Rightarrow |x^2-4| = |x-2||x+2|$$

$$< |x-2|\,4\tfrac{1}{3}$$

$$(\because |x-2| < \delta \leq \tfrac{1}{3} \Rightarrow |x+2| \leq |x-2| + 4 < 4\tfrac{1}{3})$$

$$< \varepsilon \quad (\because |x-2| < \delta \leq \tfrac{\varepsilon}{4\frac{1}{3}})$$

i.e. $\lim_{x \to 2} x^2 = 4$

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i.e. $\lim_{x \to 2} x^2 = 4$ #

If you want to write only the bare minimum.