Further study:

- de Rham Theorem: $H^k_{dR}(M) \cong \underbrace{H^k(M;\mathbb{R})}_{\text{differentiable singular cohomo.}}$

- compactly supported de Rham cohom. $H^k_c(M)$

- Thom isom & characteristic classes on vector bundles

- Hodge theory $\mathbb{H}^k(M) \to H^k_{dR}(M)$ is an isom
  $$\mathbb{H}^k(M) = \{\omega \in \Omega^k(M) \mid \Delta\omega = 0\} \text{ harmonic forms}$$
  M Riemannian, $\Delta := dd^* + d^*d : \Omega^k \to \Omega^k$ Laplacian operator
  where d^* is the operator which is (formally) adjoint to d on $\bigoplus \Omega^k(M)$ w.r.t. the inner product $(\,,\,)$.
  i.e. If $\alpha \in \Omega^{k-1}(M)$, $\beta \in \Omega^k(M)$, then
  $$(d\alpha, \beta) = (\alpha, d^*\beta) \quad [\text{adjoint}]$$
  Hodge decomposition

- Complex mfd, Dolbeault cohomology
  Kähler mfd, Hodge thm on Kähler mfd