2023 Fall, NTHU-MATH 3010: Differential Equations

Homework Assignments

9/11	§1.1:	3, 5, 7, 16, 18, 19, 20, 21, 24
	§1.3:	1, 3, 5, 6, 10, 12, 17, 19, 22
9/14	§2.1:	1, 4,, 8, 10, 11, 13, 15, 20, 23
	§2.2:	2, 5, 8, 10, 14, 18, 20, 25
9/18	§2.3:	3, 4, 6, 9, 13, 15, 20, 23
	§2.5:	1, 3, 5, 7, 16, 17, 19, 27
9/25	§2.6:	1, 3, 6, 8, 14, 16, 18, 20
	§2.7:	1, 5, 9, 11
10/02	§2.4:	9, 10, 17, 18, 19
	§2.8:	12, 13, 14, 18
10/16	§3.1:	1, 4, 7, 12, 16, 21
	§3.3:	5, 10, 15, 18, 19, 21, 23
	§3.4:	1, 6, 10, 17, 28, 29
10/19	§3.2:	4, 7, 11, 14, 16, 21, 23, 26, 29, 30
	§3.3:	25, 28
	§3.4:	24, 26, 33
10/23	§3.5:	2, 4, 7, 12, 15, 22, 26
	§3.6:	1, 3, 4, 6, 10, 12, 23, 24
11/2	§5.2:	1, 4, 8, 13, 15
	§5.3:	1, 4, 5, 9, 10, 15, 18
	§5.4:	8, 10, 17, 18
11/6	§5.5:	1, 2, 4, 10, 13, 22, 25, 27, 31
	§5.6:	1, 3, 5, 7, 10, 11
	§5.7:	2, 6, 9, 12, 15
11/9	§4.1:	5, 8, 9, 14, 15, 17
	§4.2:	9, 10, 12, 18, 20, 24, 27, 28, 29, 31
11/16	§4.3:	1, 4, 6, 9, 11, 13, 14
	§4.4:	1, 3, 6, 9, 10, 11
11/20	§6.4:	2, 4, 6, 9, 10
	§6.7:	1, 3, 7, 9, 12, 14
11/23	§6.5:	2, 6, 8, 10, 14, 21, 24
	§6.6:	1, 4, 6, 8, 11, 16, 20, 23
	§6.8:	2, 4, 9, 12, 14, 15, 19, 21
12/4	§6.9:	1, 4, 7, 12, 13
	§7.1:	6, 7, 10, 11, 13, 14, 17, 23, 24
12/7	§7.2:	2, 5, 11, 13, 16, 20
	§7.3:	11, 15, 17, 20, 23
	§7.4:	3, 7, 11, 17
	§7.6:	4, 5, 7, 9, 11, 14
12/11	§9.1:	1, 3, 11, 14, 15, 16
	§9.3:	4, 6, 9, 17, 18, 19, 23, 24, 25

12/18 §9.4: 2, 3, 6, 7
§9.5: 2, 5
§9.6: 1, 2, 4, 5, 6
12/21 §10.1: 1, 4, 7, 11, 14, 16
§11.1: 7, 9, 11, 12, 14, 16, 21
12/25 §11.2: 1, 3, 4, 7, 10, 12, 14, 15, 18, 20, 21
12/28 §11.3: 1, 4, 6, 8, 11,13, 15, 16

Here is amendment to the lecture on 12/21 about the Lyapunov stability theorem. The proof will not be part of final exam.
Ref: Hirsch-Smale's book "Differential Equations, Dynamical Systems, and Linear Algebra" §9.3

Suppose V is a strict Lyapunov function for $\dot{x} = f(x)$ on neighborhood U of isolated equilibrium point $\bar{x}$. We want to show that $\bar{x}$ is asymptotically stable.

Choose $\delta > 0$ small so that $\overline{B_\delta(\bar{x})} \subset U$. Let

$$\alpha = \inf_{\partial B_\delta(\bar{x})} V, \quad U_1 = \{x \in B_\delta(\bar{x}) : V(x) < \alpha\}.$$

Then any solution $x(t)$ starting from U_1 will stay inside $B_\delta(\bar{x})$ for $t > 0$ since $V(x(t))$ is decreasing in t. This implies stability of $\bar{x}$.

To prove asymptotic stability, we need to show that any solution $x(t)$ starting from U_1 approaches $\bar{x}$ as $t \to \infty$. Assume otherwise, then there exists some solution $x(t)$ starting from U_1 and some sequence $t_n \nearrow \infty$ such that $x(t_n) \to \bar{y} \neq \bar{x}$ as $n \to \infty$. Observe that

$$V(x(t)) \ > \ V(\bar{y}) \quad \text{for all } t > 0$$

since $V(x(t))$ is strictly decreasing in t and $\lim_{n\to\infty} V(x(t_n)) = V(\bar{y})$. Let $y(t)$ be the solution starting from $\bar{y}$, then $V(y(t)) < V(\bar{y})$ for all $t > 0$. In particular, $V(y(1)) < V(\bar{y})$. By continuous dependence on initial condition there exists δ' sufficiently small such that any solution $z(t)$ starting from $B_{\delta'}(\bar{y}) \subset U_1$ satisfies $V(z(1)) < V(\bar{y})$. Choose N large so that $x(t_N) \in B_{\delta'}(\bar{y})$, let $z(t)$ be the solution starting from $x(t_N)$, then

$$V(x(t_N + 1)) \ = \ V(z(1)) \ < \ V(\bar{y}),$$

which is a contradiction.