Real Analysis Homework 8, due 2007-10-31 in class

1. (10 points) Let $f : E \to \mathbf{R} \bigcup \{\pm\infty\}$ be a nonnegative measurable function such that $\int_E f < \infty$. Show that for any $\varepsilon > 0$ there exists $\delta > 0$ such that for any measurable subset $E_1 \subset E$ with $|E_1| < \delta$ we have $\int_{E_1} f < \varepsilon$.

2. (10 points) Do Exercise 3 in p. 85.

3. (10 points) Let $f_k : E \to \mathbf{R} \bigcup \{\pm\infty\}$ be a sequence of nonnegative measurable function satisfying $\int_E f_k \to 0$ as $k \to \infty$. Show that $f_k \to 0$ in measure as $k \to \infty$.

4. (10 points) Compute the limit

$$\lim_{n\to\infty} \int_0^{n^3} \left(1 - \frac{x}{n}\right)^n e^{x/2} dx$$

and jusify your answer.