Real Analysis Homework 7, due 2007-10-31 in class

1. (10 points)

   (a) (7 points) Do Exercise 15 in p. 62.

   (b) (3 points) Use Exercise 15 in p. 62 to prove the following: Let $f : E \to \mathbf{R} \cup \{\pm\infty\}$ be a measurable function where $|E| < \infty$ and $|f| < \infty$ a.e. on E. Show that for any $\varepsilon > 0$ there exists a constant $M > 0$ and a closed set $F \subset E$ such that $|E - F| < \varepsilon$ and

   $$|f(x)| \le M \quad \text{for all} \quad x \in F.$$

   This says that a finite function is, up to a set of small measure, a bounded function.

2. (10 points) Do Exercise 16 in p. 63.

3. (10 points) Do Exercise 18 in p. 63.

4. (20 points) Do Exercise 19 in p. 63.